QUADRATIC EQUATION

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 B

More than two roots

$a = 0, \quad b = 0, \quad c = 0$

$P^2 - 3P + 2 = 0 \Rightarrow (P-1)(P-2) = 0 \quad \Rightarrow P = 1, 2$

$\& \; P^2 - 5P + 4 = 0 \;\Rightarrow\; (P-1)(P-4) = 0 \Rightarrow P = 1,4$

$\& \; P - P^2 = 0 \Rightarrow P(1-P) = 0 \;\Rightarrow\; \& \; (P = 0, P = 1)$

common roots is $P = 1$

Sol.2 B

$(b-c)x^2 + (c-a)x + (a-b) = 0$

$x = 1$ is one root of the given equation.

we see that $\alpha\beta = \dfrac{a-b}{b-c}$

$\because \quad \alpha = 1$ (one root) $\beta = \dfrac{a-b}{b-c}$

$\Rightarrow$ roots are $\dfrac{a-b}{b-c}, 1$

Sol.3 C

$x^2 + px + q = 0 \begin{cases}\alpha \\ \beta\end{cases}$

$\& \quad x^2 + px - r = 0 \begin{cases}\gamma \\ \delta\end{cases}$

$$(\alpha-\gamma)(\alpha-\delta) = \alpha^2 - \alpha\gamma - \alpha\delta + \gamma\delta \quad \left\{\begin{array}{l}\gamma+\delta = -P \\ \gamma\delta = -r\end{array}\right.$$

$$= \alpha(\alpha - \gamma - \delta) + \gamma\delta$$

$$= \alpha^2 - (\gamma+\delta)\alpha + \gamma\delta$$

$$= \alpha^2 + p\alpha - r \quad \therefore \quad \{\alpha^2 + p\alpha + q = 0$$

$$= \alpha^2 + p\alpha + q - q - r$$

$$= -(q + r)$$

Sol.4 A

$\alpha + \beta = 3 \quad \& \quad |\alpha - \beta| = 4$

$(\alpha+\beta)^2 = 9, \quad (\alpha-\beta)^2 = 16$

$4\alpha\beta = 9 - 16 \;\Rightarrow\; \alpha\beta = \dfrac{-7}{4}$

Q.E. is $x^2 - 3x - \dfrac{7}{4} = 0 \;\Rightarrow 4x^2 - 12x - 7 = 0$

Sol.5 A

$b^2 = 4(ac + 5d^2), \quad d \in N$

$b^2 - 4ac = 20d^2$

$\Rightarrow\; b^2 - 4ac > 0 \;\; \because\; 20d^2 > 0$ always $(\because\; d \in n)$

$20d^2$ has not complete square root

$\Rightarrow \;\; \sqrt{D} = \sqrt{20d^2}$ is rational always

so roots of the equation $ax^2 + bx + c = 0$ are irrational

Sol.6 A

$ax^2 + bx + c = 0$

put $x = 2$

$4a + 2b + c = 0 \Rightarrow \quad x = 2$ is a root of equation

one roots is real $\Rightarrow$ other root is also real

$\& \quad ab > 0 \quad \Rightarrow \quad a \neq 0 \quad \Rightarrow$ both root real

Sol.7 C

$x^2 + 2x - n = 0 \qquad n \in [5, 100]$

D will be perfect square

$D = 4 + 4n = 4(1 + n)$

$\Rightarrow$ $1 + n$ is perfect square

$\Rightarrow$ $1 + n = 9, 16, 25, 36, 49, 64, 81, 100$

$\Rightarrow$ $n = 8, 15, 24, 35, 48, 63, 80, 99$

8 values

Sol.8 A

$(6x + 2)x^2 + rx + (3k - 1) = 0$

$(12k + 4)x^2 + px + (6k - 2) = 0$

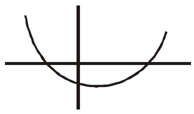
$$\frac{6k+2}{2(6k+2)} = \frac{r}{p} = \frac{3k-1}{2(3k-1)}$$

$\Rightarrow \;\; \dfrac{r}{p} = \dfrac{1}{2} \qquad \Rightarrow \;\; 2r - p = 0$

Sol.9 B

$f(x) = ax^2 + bx + c$

$\alpha + \beta = \dfrac{-b}{a} > 0, \;\; b^2 - 4\,ac > 0$



$\alpha\beta = \dfrac{c}{a} < 0$

Sol.10 B

$y = ax^2 + bx + c > 0$

if $\quad a > 0 \quad \Rightarrow \quad D < 0$

or $\quad y < 0$

if $\quad a < 0 \quad \Rightarrow \quad D < 0$

$b^2 - 4ac < 0$

Sol.11 B

$y = x^2 + kx - x + 9 > 0$

$D < 0 \quad \Rightarrow \quad (k-1)^2 - 36 < 0$

$(k + 5)(k - 7) < 0 \;\Rightarrow\; -5 < k < 7$

Sol.12 A

$ax^2 - bx + 1 = 0$

$f(x) = ax^2 - bx + 1$

$f(0) = 0 + 1 > 0 \quad \Rightarrow \quad a > 0$

$\therefore \quad f(-1) = a + b + 1 > 0$

Sol.13 C

$x^2 + ax + b = 0 \left\langle \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix} \right.$

$\Rightarrow \quad a + b = -a$

$ab = b$

$b(a - 1) = 0 \quad \Rightarrow \quad a = 1$

$\therefore \quad 2a + b = 0 \quad \Rightarrow \quad b = -2$

$x^2 + x - 2 = 0$

$\text{min value} = \frac{-D}{4a} = \frac{-9}{4}$

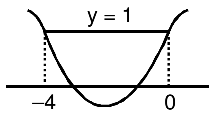
Sol.14 C

$y = -2x^2 - 6x + 9 \qquad a < 0$

$y_{max} = \frac{-D}{4a} = \frac{-108}{-8} = 13.5 \text{ at } x = \frac{-b}{2a} = \frac{-(-6)}{2(-2)} = \frac{-3}{2} = -1.5$

Sol.15 C

$f(x) = x^2 + 4x + 1$



(A) $D = 16 - 4 > 0$ real roots

(B) $x^2 + 4x + 1 > 1$

$x(x + 4) > 0$

$\Rightarrow \; x < -4$ or $x > 0$

B is wrong

(C) $f(x) \geq 1$ when correct

$x(x + 4) \geq 0 \; \Rightarrow \; x \leq -4$ or $x \geq 0$

Sol.16 D

$x^2 + 9 < (x + 3)^2 < 8x + 25$

$x^2 + 9 < x^2 + 6x + 9 \; \Rightarrow \; x > 0$

& $(x + 3)^2 < 8x + 25$

$x^2 + 6x + 9 - 8x - 25 < 0$

$x^2 - 2x - 16 < 0$

$1 - \sqrt{17} < x < 1 + \sqrt{17}$ & $x > 0$

$\Rightarrow \; x \in (0, 1 + \sqrt{17})$

Integer x = 1, 2, 3, 4, 5

No. of integer are = 5

Sol.17 B

$\Rightarrow \frac{x^2(x^2 - 3x + 2)}{(x^2 - x - 30)} \geq 0 \Rightarrow \frac{x^2(x-2)(x-1)}{(x+5)(x-6)} \geq 0$

$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

Sol.18 B

$(m - 2)x^2 + 8x + (m + 4) > 0 \quad$ for all $x \in R$

$\Rightarrow \; (m - 2) > 0 \;\&\; D < 0$

$m > 2 \;\&\; 64 - 4(m - 2)(m + 4) < 0$

$16 - m^2 - 2m + 8 < 0$

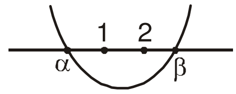
$\Rightarrow \; m^2 + 2m - 24 > 0 \quad \Rightarrow \; (m + 6)(m - 4) > 0$

$\Rightarrow \; m \in (-\infty, -6) \cup (4, \infty) \quad \therefore \; m \in (4, \infty)$

least integral value of m is 5

Sol.19 B

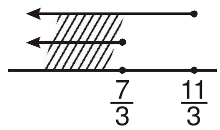
$x^2 - 2p(x - 4) - 15 = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$



$\Rightarrow \; f(1) < 0 \;\&\; f(2) < 0$

$1 + 6p - 15 < 0 \;\&\; 4 + 4p - 15 < 0$

$p < \frac{7}{3} \qquad p < \frac{11}{4}$



$\Rightarrow \; p \in \left(-\infty, \frac{7}{3}\right)$

Sol.20 D

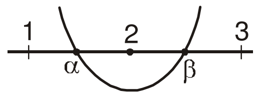
$4x^2 - 16x + \lambda = 0 \;, \quad \lambda \in R$

$1 < \alpha < 2 \;\&\; 2 < \beta < 3$

$f(1)\,f(2) < 0 \;\&\; f(2)\,f(3) < 0$

$(-12 + \lambda)(-16 + \lambda) \;\&\; (-16 + \lambda)(-12 + \lambda) = 0$

$12 < \lambda < 16$ (Integer λ) = 13, 14, 15 Three values



Sol.21 C

$x^3 - px^2 + qx - r = 0 \left\langle \begin{matrix} \alpha \\ -\alpha \\ \beta \end{matrix} \right.$

$\alpha - \alpha + \beta = p \quad \Rightarrow$ Satisfy given equation

$p^3 - p^3 + pq - r = 0$

$pq = r \qquad$ Ans.

Sol.22 B

$(x - a)(x - b)(x - c) = d \left\langle \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} \right. ; \quad d \neq 0$

Now

$(x - \alpha)(x - \beta)(x - \gamma) + d = 0$

$\Rightarrow \; (x - a)(x - b)(x - c) - d + d = 0$

$\Rightarrow \; (x - a)(x - b)(x - c) = 0 \quad$ roots are a, b, c

Sol.23 B

Let common roots is α

$$\frac{\alpha^2}{a-b} = \frac{\alpha}{2-3} = \frac{1}{3b-2a}$$

$$\alpha = \frac{a-b}{-1} \quad \& \quad \alpha = \frac{-1}{3b-2a}$$

$$\alpha = b - a \quad = \frac{1}{2a-3b}$$

$\Rightarrow$ $(b - a)(2a - 3b) = 1$

$\Rightarrow$ $5ab - 2a^2 - 3b^2 = 1$ Ans.

Sol.24 B

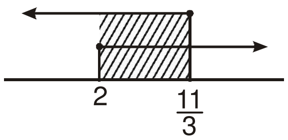
$(2 - x)(x + 1) = p$

$(x - 2)(x + 1) + p = 0$

$\Rightarrow$ $x^2 - x - 2 + p = 0$

$\frac{c}{a} > 0 \Rightarrow p - 2 > 0$

& $D > 0 \Rightarrow 1 - 4(p - 2) > 0 \Rightarrow p < \frac{9}{4}$



$p \in \left(2, \frac{9}{4}\right)$

Sol.25 A

π^x is always positive

& $-2x^2 + 6x - 9$ is always negative

$\because$ $D = 36 - 72 < 0$, leading coeff < 0

no real root $\Rightarrow$ no solution

Sol.26 B

$a > 0, b > 0, c > 0$

$ax^2 + bx + c = 0$ $D > 0$ or $= 0$ or < 0

sum of roots = $\frac{-b}{a} < 0$, product = $\frac{c}{a} > 0$

both roots have negative real parts

Sol.27 D

$$\left(\frac{1}{2}\right)^{x^2-2x} < \left(\frac{1}{2}\right)^2 \quad \Rightarrow \quad -(x^2 - 2x) < -2$$

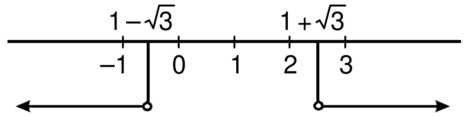
$\Rightarrow$ $x^2 - 2x > 2$ $\Rightarrow$ $x^2 - 2x - 2 > 0$

$$\alpha, \beta = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

$\alpha = 1 - \sqrt{3}$, $\beta = 1 + \sqrt{3}$

$(x - \alpha)(x - \beta) > 0$

$x \in \left(-\infty, 1 - \sqrt{3}\right) \cup \left(1 + \sqrt{3}, \infty\right)$



x can be in $(3, \infty)$

Sol.28 C

$$y = \frac{2x}{1+x^2}, x \in R$$

$\Rightarrow$ $yx^2 - 2x + y = 0$

$\Rightarrow$ $D \geqslant 0$ $\Rightarrow$ $4 - 4y^2 \geq 0$

$\Rightarrow$ $(y^2 - 1) \leqslant 0$ $\Rightarrow$ $y \in [-1, 1]$

$\therefore$ Range of $f(y) = y^2 + y - 2$

Min value = $\frac{-D}{4a} = \frac{-9}{4}$ at $y = \frac{-b}{2a} = \frac{-1}{2}$

$y = \frac{-1}{2} \in [-1, 1]$

$f(-1) = 1 - 1 - 2 = -2$

$f(1) = 1 + 1 - 2 = 0$

max value is = 0

Range $\left[\frac{-9}{4}, 0\right]$

Sol.29 C

$x^2 + 2xy + 2y^2 + 4y + 7$

$= x^2 + 2xy + y^2 + y^2 + 4y + 7$

$= (x + y)^2 + y^2 + 4y + 7$

$= (x + y)^2 + (y + 2)^2 + 3$

least value of $(x + y)^2$ is 0 when $y = -x$

least value of $(y + 2)^2$ is 0 when $y = -2$

$\therefore$ least value is = 0 + 0 + 3 = 3 Ans.

Sol.30 D

$x^2 + 2(k - 1)x + k + 5 = 0$

Case - I

(i) $D \geqslant 0$

$\Rightarrow$ $4(k - 1)^2 - 4(k + 5) \geqslant 0$

$\Rightarrow$ $k^2 - 3k - 4 \geqslant 0 \Rightarrow (k + 1)(k - 4) \geqslant 0$

$\Rightarrow$ $k \in (-\infty, -1] \cup [4, \infty)$

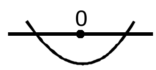
& (ii) $f(0) > 0 \Rightarrow k + 5 > 0 \Rightarrow k \in (-5, \infty)$

& (iii) $\frac{-b}{2a} > 0$ $\Rightarrow$ $\frac{-2(k-1)}{2} > 0$

$\Rightarrow k \in (-\infty, 1)$ $\therefore$ $k \in [-5, -1]$

Case - II $f(0) \leq 0 \Rightarrow k + 5 \leq 0$

$\Rightarrow k \in (-\infty, -5]$



Finally $k \in$ (Case - I) $\cup$ (Case - II)

$k \in (-\infty, -1]$

Sol.31 A

$$\frac{6x^2-5x-3}{x^2-2x+6} \leqslant 4$$

D^r is always > 0

$6x^2 - 5x - 3 - 4x^2 + 8x - 24 \leqslant 0$

$\Rightarrow\ 2x^2 + 3x - 27 \leqslant 0$

$\Rightarrow\ (2x+9)(x-3) \le 0 \quad \Rightarrow\ x \in \left[\frac{-9}{2},\ 3\right]$

least value of $4x^2 = 4.0^2 = 0$

Highest value of $4x^2$ is $= \max\left(4\cdot\left(-\frac{9}{2}\right)^2,\ 4\cdot3^2\right)$

$= \max(81, 36) = 81$

Sol.32 B

$x^4 - Kx^3 + Kx^2 + Lx + M = 0$ with roots $\alpha, \beta, \gamma, \delta$

$\alpha + \beta + \gamma + \delta = K$

$\Sigma\alpha\beta = K$

$(\alpha + \beta + \gamma + \delta)^2 = K^2$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2\Sigma\alpha\beta = K^2$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = K^2 - 2K = (K-1)^2 - 1$

minimum value $= -1$ at $K = 1$

Sol.33 A

$x^3 + Px^2 + Qx - 19 = 0$ with roots $\alpha+1, \beta+1, \gamma+1$

$(\alpha + 1)(\beta + 1)(r + 1) = 19$

$\Rightarrow\ (\alpha + \beta + r) + (\alpha\beta + \beta r + r\alpha) + (\alpha\beta r) + 1 = 19$

$x^3 - Ax^2 + Bx - C = 0$ with roots α, β, r

$\alpha + \beta + \gamma = A,\quad \Sigma\alpha\beta = B,\quad \Sigma\alpha\beta\gamma = C$

$\Rightarrow\ A + B + C + 1 = 19 \quad \Rightarrow\ A + B + C = 18$

Sol.34 A

$x^3 + 5x^2 + px + q = 0$ with roots α, β, x_1

$(\alpha + \beta) + x_1 = -5 \quad \ldots$ (i)

$\alpha\beta + (\alpha + \beta)x_1 = p$

$x^3 + 7x^2 + px + r = 0$ with roots α, β, x_2

$(\alpha + \beta) + x_2 = -7 \quad \ldots$ (ii)

$\alpha\beta + (\alpha + \beta)x_2 = p$

$(\alpha + \beta)(x_1 - x_2) = 0$

$x_1 \neq x_2 \quad \therefore\ \alpha + \beta = 0$

Put in (i) & (ii)

$x_1 = -5 \quad x_2 = -7$

Sol.35 B

$ax^2 + bx + c = 0, \quad a \neq 0,$

Given

$a + b + c < 0 \quad \& \quad D < 0$

$\Rightarrow\ f(x) < 0\ \forall x \in R \quad \Rightarrow\ f(-2) < 0$

$\Rightarrow\ 4a - 2b + c < 0 \quad \Rightarrow\ 4a + c < 2b$

Sol.36 B

$(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x - 1 < 0$ for $\forall x \in R$

Case - I

$D < 0$

$\Rightarrow\ (\lambda + 2)^2 + 4(\lambda^2 + \lambda - 2) < 0$

$\Rightarrow\ \lambda^2 + 4\lambda + 4 + 4\lambda^2 + 4\lambda - 8 < 0$

$\Rightarrow\ 5\lambda^2 + 8\lambda - 4 < 0$

$\Rightarrow\ (\lambda + 2)(5\lambda - 2) < 0 \quad \Rightarrow\ \lambda \in \left(-2, \frac{2}{5}\right)$

& $\lambda^2 + \lambda - 2 < 0 \quad \Rightarrow (\lambda + 2)(\lambda - 1) < 0$

$\lambda \in (-2, 1)$

$\therefore\ \lambda \in \left(-2, \frac{2}{5}\right)$

Case - II

If $\lambda^2 + \lambda - 2 = 0$

$(\lambda + 2)(\lambda - 1) = 0$

$\Rightarrow\ \lambda = -2, 1$

for $\lambda = -2$ Satisfy the given in equaility

$0x^2 + 0x - 1 < 0$

$\Rightarrow\ \lambda \in \left[-2, \frac{2}{5}\right)$

Sol.37 D

$x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ with roots α, β

$\alpha + \beta < 1 \quad \& \quad \alpha\beta < 1$

$\lambda^2 - 5\lambda + 5 < 1 \quad \& \quad 2\lambda^2 - 3\lambda - 4 < 1$

$\lambda^2 - 5\lambda + 4 < 0 \quad \& \quad 2\lambda^2 - 3\lambda - 5 < 0$

$(\lambda - 4)(\lambda - 1) < 0 \quad \& \quad (2\lambda - 5)(\lambda + 1) < 0$

$\lambda \in (1, 4) \quad \& \quad \lambda \in \left(-1, \frac{5}{2}\right)$

$\Rightarrow \lambda \in \left(1, \frac{5}{2}\right)$

Sol.38 A

$C_1 : b^2 - 4ac \geqslant 0,$

$ax^2 + bx + c = 0$ real roots

C_1 satisfied

C_2 : a, –b, c are some sign

$\alpha + \beta > 0 \quad \Rightarrow \quad \frac{-b}{a} > 0$

$\alpha\beta > 0 \quad \Rightarrow \quad \frac{c}{a} > 0$

C_2 satisfied

C_1 & C_2 are satisfied

Sol.39 B

$x^2 + 2ax + b = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$D > 0 \qquad |\alpha - \beta| \leqslant 2m$

$4a^2 - 4b > 0$

$a^2 - b > 0$

$\Rightarrow \; b < a^2, \qquad \alpha + \beta = -2a, \; \alpha\beta = b$

$|\alpha - \beta|^2 \leqslant (2m)^2$

$(-2a)^2 - 4(b) \leqslant 4m^2$

$a^2 - b \leqslant m^2$

$b \geqslant a^2 - m^2$

$b \in [a^2 - m^2, a^2)$

Sol.40 A

$x^2 + ax + 1 = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$|\alpha - \beta| < \sqrt{5} \quad \Rightarrow \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\Rightarrow \quad \sqrt{a^2 - 4} < \sqrt{5} \;\Rightarrow\; a^2 < 9 \quad \Rightarrow \quad a \in (-3, 3)$

& $a^2 - 4 \geq 0 \;\Rightarrow\; a \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow \quad a \in (-3, -2] \cup [2, 3)$

Sol.41 A

$x^2 + px + q = 0$

$\tan 30º + \tan 15º = -p \quad \& \tan 30º \cdot \tan 15º = q$

$\tan 45º = \frac{\tan 30º + \tan 15º}{1 - \tan 30º \tan 15º} = \frac{-p}{1 - q}$

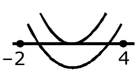
$\Rightarrow \quad 1 - q = -p \quad \Rightarrow \quad q - p = 1 \;\Rightarrow\; 2 + q - p = 3$

Sol.42 B

$x^2 - 2mx + m^2 - 1 = 0$

(i) $D \geqslant 0$

$4m^2 - 4(m^2 - 1) \geqslant 0$



$\Rightarrow \quad 4 \geqslant 0 \;\Rightarrow\; m \in R$

& (ii) $f(-2) > 0$

$4 + 4m + m^2 - 1 > 0 \;\Rightarrow\; m^2 + 4m + 3 > 0$

$(m + 3)(m + 1) = 0 \Rightarrow m \in (-\infty, -3) \cup (-1, \infty)$

& (iii) $f(4) > 0$

$16 - 8m + m^2 - 1 > 0$

$m^2 - 8m + 15 > 0$

$(m - 3)(m - 5) > 0$

$\Rightarrow \qquad m \in (-\infty, 3) \cup (5, \infty)$

& (iv) $\quad -2 < \frac{-b}{2a} < 4$

$-2 < \frac{2m}{2} < 4 \quad \Rightarrow \quad m \in (-2, 4)$

finally $m \in (-1, 3)$

Sol.43 D

$x^2 - (a - 2)x - a - 1 = 0 \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

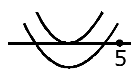
$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6$

Min $(\alpha^2 + \beta^2)$ at $\frac{-B}{2A} = \frac{+2}{2} = 1 \quad \Rightarrow \quad a = 1$

Sol.44 B

$x^2 - 2kx + k^2 + k - 5 = 0$



(i) $D \geqslant 0$

$4k^2 - 4k^2 - 4k + 20 \geqslant 0 \;\Rightarrow\; k < 5$

& (ii) $f(5) > 0$

$25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow \; k^2 - 9k + 20 > 0 \;\Rightarrow (k - 4)(k - 5) > 0$

$\Rightarrow \; k \in (-\infty, 4) \cup (5, \infty)$

& (iii) $\frac{-b}{2a} < 5 \quad \Rightarrow \quad \frac{2k}{2} < 5 \quad \Rightarrow \quad k < 5$

$\therefore$ finally $k \in (-\infty, 4)$

Sol.45 C

$x^2 + px + (1 - p) = 0$

$(1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0 \quad \Rightarrow \quad p = 1$

Q.E. will be $\Rightarrow \; x^2 + x = 0 \;\Rightarrow\; x(x + 1) = 0$

$\Rightarrow \; x = 0, -1$

Aliter

$\alpha + 1 - p = -p \qquad \Rightarrow \quad \alpha = -1$

Satisfies

$1 - p + 1 - p = 0 \qquad \Rightarrow \quad p = 1$

$\beta = 1 - p = 0 \qquad \Rightarrow \quad \beta = 0$

Sol.46 A

$x^2 + px + 12 = 0$

$\Rightarrow \; 4^2 + p4 + 12 = 0 \;\Rightarrow 4p = -28 \;\Rightarrow p = -7$

Now second equation

$\Rightarrow \; x^2 - 7x + q = 0$ has equal roots

$\Rightarrow$ D = 0 $\Rightarrow$ 49 – 4q = 0 $\Rightarrow$ q = $\dfrac{49}{4}$

Sol.47 C

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow (\alpha + \beta) = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \Rightarrow \frac{-b}{a} = \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2}{c^2} - \frac{2a}{c} \quad \Rightarrow \quad \frac{2a}{c} = \frac{b^2}{c^2} + \frac{b}{a}$$

$$\Rightarrow \frac{2a}{c} = \frac{ab^2 + bc^2}{ac^2} \Rightarrow \frac{2a^2c}{abc} = \frac{ab^2}{abc} + \frac{bc^2}{abc}$$

$$\Rightarrow \frac{2a}{b} = \frac{b}{c} + \frac{c}{a} \quad \Rightarrow \quad \frac{2}{\left(\frac{b}{a}\right)} = \frac{1}{\left(\frac{c}{b}\right)} + \frac{1}{\left(\frac{a}{c}\right)}$$

$\Rightarrow \dfrac{a}{c}, \dfrac{b}{a}, \dfrac{c}{b}$ in H.P.

Sol.48 A

$$\alpha + 2\alpha = \frac{-(3a-1)}{(a^2 - 5a + 3)} \;\&\; 2\alpha^2 = \frac{2}{(a^2 - 5a + 3)}$$

$$\Rightarrow \alpha^2 = \frac{(3a-1)^2}{9(a^2 - 5a + 3)^2} = \frac{1}{(a^2 - 5a + 3)}$$

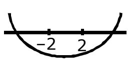
$\Rightarrow$ $9a^2 - 6a + 1 = 9a^2 - 45a + 27$

$\Rightarrow$ 39a = 26 $\Rightarrow$ a = $\dfrac{2}{3}$

Sol.49 D

$ax^2 + bx + c = 0$ a, b, c $\in$ R

$\Rightarrow x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0$



$\Rightarrow f(x) = x^2 + \dfrac{b}{a}x + \dfrac{c}{a}$

f(–2) < 0 & f(+2) < 0

4a – 2b + c < 0 4a + 2b + c < 0

$4 - \dfrac{2b}{a} + \dfrac{c}{a} < 0$ $4 + \dfrac{2b}{a} + \dfrac{c}{a} < 0$

Sol.50 B

$a^2x^4 + bx^3 + cx^2 + dx + f^2$ is perfect square

in the form

$= (ax^2 + mx + f)^2$

$= a^2x^4 + (2\,am)x^3 + (m^2 + 2af)x^2 + 2mfx + f^2$

by comparision

2am = b , $c = m^2 + 2af$

$m = \dfrac{b}{2a}$, $c = \dfrac{b^2}{4a^2} + 2af$

$4a^2c = b^2 + 8a^3f$

$4a^2c - b^2 = 8a^3f$

Sol.51 B

$a \leqslant 0$ $x^2 - 2a\,|x - a| - 3a^2 = 0$

If x = a $\Rightarrow$ $a^2 - 3a^2 = 0$ $\Rightarrow$ a = 0

x > a

$\Rightarrow$ $x^2 - 2ax - a^2 = 0$

$$x = \frac{2a \pm \sqrt{4a^2 + 4a^2}}{2} = a \pm \sqrt{2}a$$

$a + \sqrt{2}a < a$ $\Rightarrow$ $x \neq a + \sqrt{2}a$

or $a - \sqrt{2}a > a$ $\therefore$ **x = (1 – $\sqrt{2}$)a**

x < a

$x^2 + 2ax - 5a^2 = 0$

$$x = \frac{-2a \pm \sqrt{4a^2 + 20a^2}}{2} = \frac{-2a \pm 2\sqrt{6}a}{2}$$

$x = -a \pm \sqrt{6}a$

$x \neq -a(1 + \sqrt{6})$ ($\because$ x < 0)

or $-a + \sqrt{6}a < a$ $\therefore$ **x = (–1 + $\sqrt{6}$)a**

Sol.52 B

$x^2 - xy + y^2 - 4x - 4y + 16 = 0$ x, y $\in$ R

$x^2 - x(y + 4) + (y^2 - 4y + 16) = 0$ … (1)

x $\in$ R $\Rightarrow$ D $\geqslant$ 0

$(y + 4)^2 - 4(y^2 - 4y + 16) \geqslant 0$

$\Rightarrow$ $y^2 + 8y + 16 - 4y^2 + 16y - 64 \geqslant 0$

$\Rightarrow$ $y^2 - 8y + 16 \leqslant 0$

$\Rightarrow$ $(y - 4)^2 \leqslant 0$ $\Rightarrow$ y = 4

Put is given equation (i)

$x^2 - 8x + 16 = 0$

$\Rightarrow$ $(x - 4)^2 = 0$ $\Rightarrow$ x = 4

Sol.53 D

$x^4 - 4x^3 + ax^2 + bx + 1 = 0$

real & positive roots

$\alpha + \beta + r + \delta = 4$ & $\alpha\beta r\delta = 1$

$\Rightarrow$ $\alpha = \beta = r = \delta = 1$

$\Sigma\alpha\beta = a$ $\Rightarrow$ a = 6

$\Sigma\alpha\beta r = -b$ $\Rightarrow$ b = –4

or $(x - 1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

Sol.54 C

$ax^2 + bx + c = 0 \qquad \alpha + \beta = \frac{-b}{a}, \qquad \alpha\beta = \frac{c}{a}$

$\alpha^3 + \beta^3 = (\alpha + \beta)\,[(\alpha + \beta)^2 - 3\alpha\beta]$

$$\alpha^3 + \beta^3 = \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\frac{c}{a}\right]$$

$$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right] = \frac{-b}{a}\frac{(b^2 - 3ac)}{a^2} = \frac{3abc - b^3}{a^3}$$

Sol.55 D

$(y - 1)x^2 + (y + 1)x + (2cy - c) = 0$

$D \geqslant 0 \qquad \therefore\ x \in R$

$\Rightarrow\ (y + 1)^2 - 4(y - 1)(2cy - c) \geqslant 0$

$y^2 + 2y + 1 - 8cy^2 + 12cy - 4c \geqslant 0$

$(1 - 8c)y^2 + (2 + 12c)y + (1 - 4c) \geqslant 0$

$\forall\ y \in R \qquad D \leqslant 0$

$(2 + 12c)^2 - 4(1 - 8c)(1 - 4c) \leqslant 0$

$(1 + 6c)^2 - (1 - 8c)(1 - 4c) \leqslant 0$

$4c^2 + 24c \leqslant 0 \quad \Rightarrow \quad c \in [-6, 0]$

& N^r & D^r have no any common root

(i) both common factor (root) (not possible)

$$\frac{1}{1} = \frac{-1}{+1} = \frac{c}{2c}$$

(ii) If one common root is α

$(\alpha^2 - \alpha + c = 0) \times 2$

& $\quad \alpha^2 + \alpha + 2c = 0$

$\alpha^2 - 3\alpha = 0$

$\alpha = 0 \qquad \Rightarrow \qquad c = 0$

or $\ \alpha = 3 \qquad \Rightarrow \qquad c = -6$

$\therefore\ c \neq 0 \qquad \& \qquad c \neq -6$

$\therefore\ c \in (-6, 0)$

Sol.56 B

$mx^2 - 9mx + 5m + 1 > 0 \qquad \forall\ x \in R$

$D < 0 \qquad \& \qquad m > 0$

$81\,m^2 - 4m\,(5m + 1) < 0$

$81\,m^2 - 20m^2 - 4m < 0$

$61m^2 - 4m < 0$

$$m(61m - 4) < 0 \ \Rightarrow\ m \in \left(0, \frac{4}{61}\right)$$

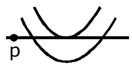
If $m = 0 \ \Rightarrow\ 1 > 0 \qquad \forall\ x \in R \ \Rightarrow\ m = 0$

$$\mathbf{m \in \left[0, \frac{4}{61}\right)}$$

Sol.57 B

(i) $D \geqslant 0$

$1 - 4p \geqslant 0 \ \Rightarrow\ p \leq \frac{1}{4}$



& (ii) $f(p) > 0$

$p^2 + p + p > 0 \quad \Rightarrow \quad p(p + 2) = 0$

$\Rightarrow\ p \in (-\infty, -2), \cup (0, \infty)$

& (iii) $\frac{-b}{2a} > p \qquad \Rightarrow \quad -\frac{1}{2} > p$

finally $p \in (-\infty, -2)$

Sol.58 A

$2a^2x^2 - 2abx + b^2 = 0$

$D = 4a^2b^2 - 8a^2b^2$

$= -4a^2b^2 < 0$

$\Rightarrow$ roots are non real

$p^2x^2 + 2pq + q^2 = 0$

$D = 4p^2q^2 - 4p^2q^2 = 0$

equal real roots

no common roots of given equations.

Sol.59 C

$a^2 + b^2 + c^2 = 1$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2\Sigma ab$

$= 1 + 2\Sigma ab$

$$\Sigma ab = \frac{(a + b + c)^2 - 1}{2} \Rightarrow \text{Min } \Sigma ab = \frac{0 - 1}{2} = -\frac{1}{2}$$

Now $\ \frac{1}{2}\left[(a - b)^2 + (b - c)^2 + (c - a)^2\right] \geqslant 0$

$a^2 + b^2 + c^2 - ab - bc - ca \geqslant 0 \ \Rightarrow\ 1 \geqslant \Sigma ab$

$$\therefore\ \Sigma ab \in \left[-\frac{1}{2}, 1\right]$$

Sol.60 C

$3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0$

$f(0) < 0 \qquad \Rightarrow k^2 - 3k + 2 < 0$

$\Rightarrow\ (k - 2)(k - 1) < 0 \quad \Rightarrow\ k \in (1, 2)$

Sol.61 D

$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

sum of roots $= (2\alpha + 3\beta) + (3\alpha + 2\beta)$

$$= 5(\alpha + \beta) = 5\left(-\frac{b}{a}\right)$$

Product of roots $= 6\alpha^2 + 6\beta^2 + 13\alpha\beta$

$= 6(\alpha + \beta)^2 + \alpha\beta$

$$= 6\left(\frac{-b}{a}\right)^2 + \frac{c}{a} = \frac{6b^2}{a^2} + \frac{c}{a}$$

Q. E. $x^2 + \frac{5b}{a}x + \frac{6b^2}{a^2} + \frac{c}{a} = 0$

$a^2x^2 + 5abx + 6b^2 + ac = 0$

Sol.62 B

$(a-1)(x^2+x+1)^2 - (a+1)(x^4+x^2+1) = 0$

$(a-1)(x^2+x+1)^2$

$- (a+1)(x^2+x+1)(x^2-x+1) = 0$

$(x^2+x+1)[(a-1)(x^2+x+1)$

$- (a+1)(x^2-x+1)] = 0$

$x^2 + x + 1 > 0 \quad \forall x \in R$

$\Rightarrow (a-1)(x^2+x+1) - (a+1)(x^2-x+1) = 0$

$\Rightarrow -2x^2 + 2ax - 2 = 0 \quad \Rightarrow x^2 - ax + 1 = 0$

$D > 0 \Rightarrow a^2 - 4 > 0$

$a \in (-\infty, -2) \cup (2, \infty)$

Sol.63 A

$$\Rightarrow \frac{(2x-1)}{x(2x^2+3x+1)} > 0 \quad \Rightarrow \frac{(2x-1)}{x(x+1)(2x+1)} > 0$$

$x \in (-\infty, -1) \cup \left(\frac{-1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$ contains $\left(-\infty, \frac{3}{2}\right)$

Sol.64 B

Case - I $\quad b > 0 \quad \Rightarrow ax^2 + 2bx + b > 0$

$a > 0, D < 0$

$4b^2 - 4ab < 0$

$(b^2 - ab) < 0$

Case - II $\quad b < 0 \quad \Rightarrow ax^2 + 2bx + b < 0$

$a < 0, D < 0$

$4b^2 - 4ab < 0$

$b^2 - ab < 0$

In both case $(b^2 - ab) < 0$

Now $bx^2 + (b-c)x + b - c - a = 0$

$D = (b-c)^2 - 4b(b-c-a)$

$D = b^2 + c^2 - 2bc - 4b^2 + 4bc + 4ab$

$D = (b+c)^2 - 4(b^2 - ab)$

$(b+c)^2 > 0 \quad \& \quad (b^2 - ab) < 0$

$\Rightarrow D > 0 \quad \Rightarrow$ real & Distinct root

Sol.65 B

Let $\quad ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a} = \alpha^2 + \beta^2$

$\Rightarrow (\alpha + \beta)[(\alpha + \beta) - 1] = 2\alpha\beta$

$\alpha\beta = \frac{c}{a} = \alpha^2\beta^2$

$\alpha\beta(\alpha\beta - 1) = 0$

$\Rightarrow \alpha\beta = 0 \quad$ or $\quad \alpha\beta = 1$

If $\alpha\beta = 0$

$(\alpha + \beta)[(\alpha + \beta) - 1] = 0$

$\alpha + \beta = 0, \quad \alpha + \beta = 1$

If $\alpha\beta = 1$

$(\alpha + \beta)^2 - (\alpha + \beta) - 2 = 0$

$((\alpha + \beta) - 2)((\alpha + \beta) + 1) = 0$

$\alpha + \beta = 2 \quad \alpha + \beta = -1$

Quadratic Equations are

(1) $x^2 + 0x + 0 = 0$ (2) $x^2 - x + 0 = 0$

(3) $x^2 - 2x + 1 = 0$ (4) $x^2 + x + 1 = 0$

EXERCISE – II HINTS & SOLUTIONS

Sol.1 B,D

$ax^2 + bx + c = 0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix} \Rightarrow \alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

& $px^2 + qx + r = 0 \begin{smallmatrix}\alpha+h\\ \beta+h\end{smallmatrix}$

$\alpha + \beta + 2h = \frac{-q}{p} \quad \Rightarrow \quad -\frac{b}{a} + 2h = -\frac{q}{p}$

$\Rightarrow \quad h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

& $(\alpha + h)\,(\beta + h) = \frac{r}{p} \Rightarrow \alpha\beta + h\,(\alpha + \beta) + h^2 = \frac{r}{p}$

$\Rightarrow \quad \frac{c}{a} + h\left(-\frac{b}{a}\right) + h^2 = \frac{r}{p}$

$\Rightarrow \quad \frac{c}{a} - \frac{b}{2a}\left(\frac{b}{a} - \frac{q}{p}\right) + \frac{1}{4}\left(\frac{b}{a} - \frac{q}{p}\right)^2 = \frac{r}{p}$

$\Rightarrow \quad \frac{c}{a} - \frac{b^2}{2a^2} + \frac{bq}{2ap} + \frac{1}{4}\frac{b^2}{a^2} - \frac{bq}{2ap} + \frac{1}{4}\frac{q^2}{p^2} = \frac{r}{p}$

$\Rightarrow \frac{c}{a} - \frac{b^2}{4a^2} - \frac{r}{p} - \frac{1}{4}\frac{q^2}{p^2} \quad \Rightarrow \quad \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

Sol.2 B,C,D

$x^2 + ab + b = 0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}, \qquad a, b \neq 0 \in R$

(A) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (a^2 - 2b)$
$\alpha^2\beta^2 = b^2 \quad \Rightarrow \quad x^2 - (a^2 - 2b)x + b^2$

(B) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-a}{b}$ & $\frac{1}{\alpha\beta} = \frac{1}{b}$

$\Rightarrow \quad x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow \quad bx^2 + ax + 1 = 0$

(C) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{a^2 - 2b}{b}$ & $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

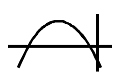
$\Rightarrow x^2 - \frac{(a^2 - 2b)}{b}x + 1 \quad \Rightarrow \quad bx^2 + (2b - a^2)x + b = 0$

(D) $(\alpha - 1) + (\beta - 1) = \alpha + \beta - 2$
$= -a + 2 = -(a + 2)$
$(\alpha - 1)\,(\beta - 1) = \alpha\beta - (\alpha + \beta) + 1$
$= b + a + 1$
$x^2 + (a + 2)x + b + a + 1 = 0$

Sol.3 A,B,C,D

$y = ax^2 + bx + c$

(A)



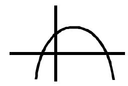
$a < 0$

$-\frac{b}{a} < 0 \Rightarrow b < 0$

$\frac{c}{a} > 0 \Rightarrow c < 0$

$abc < 0$

(B)



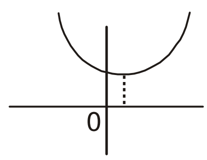
$a < 0$

$-\frac{b}{a} > 0 \Rightarrow b > 0$

$\frac{c}{a} < 0 \Rightarrow c > 0$

$abc < 0$

(C)

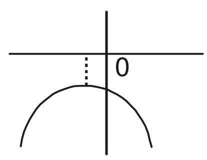


$a > 0$ & $c > 0$

$\frac{-b}{2a} > 0 \Rightarrow b < 0$

$abc < 0$

(D)



$a < 0$ & $c < 0$

$\frac{-b}{2a} < 0 \Rightarrow b < 0$

$abc < 0$

Sol.4 A,D

$ax^3 + bx^2 + cx + d = 0$
have a factor $x^2 + x + 1$
one root real & 2 Imaginary α, β, γ

$\alpha + \beta + r = \frac{-b}{a} \qquad \beta + r = -1,\ \beta\gamma = 1$

$\alpha - 1 = \frac{-b}{a} \qquad \Rightarrow \alpha = \frac{a - b}{a}$

or $\alpha(\beta\, r) = \frac{-d}{a} \qquad \Rightarrow \qquad \alpha = \frac{-d}{a}$

Sol.5 B,D

Let a common root is α

$\alpha^2 + ab\alpha + c = 0$
$\alpha^2 + ac\alpha + b = 0$

$\alpha\ a\,(b - c) = b - c$

If $b = c \Rightarrow$ both are roots are common $\therefore\ \alpha = \frac{1}{a}$

& $\alpha\beta = c \qquad \alpha\gamma = b$
$\beta = ac \qquad \gamma = ab$
$\beta + \gamma = a\,(b + c)$ & $\beta\gamma = a^2bc$

Q.E. is $x^2 - a(b + c)x + a^2bc = 0$

further $\frac{1}{a}$ satisfied given equation

$$\frac{1}{a^2} + a \cdot b \cdot \frac{1}{a} + c = 0$$

$\Rightarrow$ $1 + a^2(b + c) = 0$

$\Rightarrow$ $a(b + c) = \frac{-1}{a}$

$\therefore$ $x^2 - a(b + c)x + a^2bc = 0$

$\Rightarrow$ $\frac{1}{a}x^2 - (b + c)x + abc = 0$

$\Rightarrow$ $a(b + c)x^2 + (b + c)x - abc = 0$

Sol.6 C,D

$ax^2 + bx + c = 0$

$x^2 + 4x + 5 = 0$ non real root

$\therefore$ both roots are common

$b^2 - 4ac < 0$

$\frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$ $\Rightarrow$ $a = k, b = 4k, c = 5k$

$k \in R$, $k \neq 0$

If $a > 0$ $\Rightarrow$ $c > 0$

but b may be (+) or (–)

& If $a < 0$ $\Rightarrow$ $c < 0$

but b may be (+) or (–)

Sol.7 A,D

$x^2 + px + q = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$D > 0$ $\Rightarrow$ $p^2 - 4q > 0$

& $\alpha + \beta = -p$, $\alpha\beta = q$

& $x^2 - rx + s = 0 \begin{cases} \alpha^4 \\ \beta^4 \end{cases}$

$\alpha^4 + \beta^4 = r$, $\alpha^4\beta^4 = s$

Now $x^2 - 4qx + 2q^2 - r = 0$

$D = 16q^2 - 4(2q^2 - r)$

$= 4(4q^2 - 2q^2 + r) = 4(2q^2 + r)$

$= 4(2\alpha^2\beta^2 + \alpha^4 + \beta^4)$

$= 4(\alpha^2 + \beta^2)^2 > 0$

$\Rightarrow$ two roots are real

Product of roots $= 2q^2 - r$

$= 2\alpha^2\beta^2 - (\alpha^4 + \beta^4)$

$= -(\alpha^2 - \beta^2)^2 < 0$

one is positive & other is negative

Sol.8 A,D

$20x^2 + 210x + 400 = 4500 \Rightarrow 2x^2 + 21x - 410 = 0$

$\Rightarrow$ $(2x + 41)(x - 10) = 0$

$\Rightarrow$ $x = \frac{-41}{2}$, $x = 10$ $\Rightarrow$ $x = -20.5$, $x = 10$

Sol.9 A,B,C,D

$x^3 + bx^2 + cx - 1 = 0 \begin{cases} \alpha = \frac{a}{r} \\ \beta = a \\ r = ar \end{cases}$

$\frac{a}{r} + a + ar = -b$ $\Rightarrow$ $a\left(\frac{1}{r} + 1 + r\right) = -b$

& $\frac{a}{r} \times a \times ar = 1$

$a^3 = 1$ $\Rightarrow$ $a = 1$

& $\frac{a}{r}a + a \cdot ar + \frac{a}{r} \cdot ar = c$

$a^2\left(\frac{1}{r} + r + 1\right) = c$

$\frac{1}{r} + r + 1 = -b$ & $\frac{1}{r} + r + 1 = c$ $\Rightarrow$ $b + c = 0$

we know $\frac{1}{r} + r > 2$ $\Rightarrow$ $\left(\frac{1}{r} + r + 1\right) > 3$

$-b > 3$ $\Rightarrow$ $b < -3$ $\Rightarrow$ $b \in (-\infty, -3)$

& other two roots are $\frac{1}{r}$ & r

if $\frac{1}{r} > 1$ $\Rightarrow$ $r < 1$

if $r > 1$ $\Rightarrow$ $r < 1$

Sol.10 A,B

$$f(x) = \frac{3}{(x-2)} + \frac{4}{(x-3)} + \frac{5}{(x-4)} = 0$$

$6x^2 - 14x - 21x + 49 = 0$

$(3x - 7)(2x - 7) = 0$

$x = \frac{7}{3}$, $x = \frac{7}{2}$

$2 < \frac{7}{2} < 3$ $3 < \frac{7}{2} < 4$

Aliter

$g(x) = 3(x - 3)(x - 4) + 4(x - 2)(x - 4) + 5(x - 2)(x - 3) = 0$

$g(2) > 0$; $g(3) < 0$, $g(4) > 0$

one root lie b/w (2, 3)

& other root lie b/w (3, 4)

EXERCISE – III HINTS & SOLUTIONS

Sol.1 $(a^2 - a - 2)x^2 + (a^2 - 4)x + (a^2 - 3a + 2) = 0$

more than two sol. $\Rightarrow$ an identity

$a^2 - a - 2 = 0 \Rightarrow (a-2)(a+1) = 0$

$a^2 - 4 = 0 \Rightarrow (a-2)(a+2) = 0$

$a^2 - 3a + 2 = 0 \Rightarrow (a-2)(a-1) = 0$

$\therefore a = 2$

$(x^2 + x + 1)a^2 + (-x^2 - 3)a + (-2x^2 - 4x + 2) = 0$

$x^2 + x + 1 \neq 0$

$\Rightarrow$ No, there exist any real x for equation an identity in a

Sol.2 $ax^2 + bx + c = 0 \begin{matrix}\diagup \alpha \\ \diagdown \beta\end{matrix}$

(i) $a + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$

$\Rightarrow$ sum $= \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta}$

$= (\alpha+\beta) + \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-b}{a}\left(\frac{a+c}{c}\right)$

Product $= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \frac{(\alpha\beta+1)}{\alpha\beta} = \frac{(c+a)^2}{ac}$

Q.E. is $x^2 + \frac{b(a+c)x}{ac} + \frac{(a+c)^2}{ac} = 0$

$\Rightarrow acx^2 + b(a + c)x + (a + c)^2 = 0$

(ii) $(\alpha^2 + 2), (\beta^2 + 2)$

sum $= \alpha^2 + \beta^2 + 4 = (\alpha + \beta)^2 - 2\alpha\beta + 4$

$= \frac{b^2 - 2ac + 4a^2}{a^2}$

Product $= (\alpha\beta)^2 + 2(\alpha^2 + \beta^2) + 4$

$= \frac{c^2}{a^2} + 2\left(\frac{b^2}{a^2} - 2\frac{c}{a}\right) + 4$

$= \frac{c^2 + 2b^2 - 4ac + 4a^2}{a^2} = \frac{2b^2 + (c-2a)^2}{a^2}$

Q.E. is

$\Rightarrow a^2x^2 + (2ac - b^2 - 4a^2)x + 2b^2 + (c - 2a)^2 = 0$

Sol.3 $\alpha^2 = 5\alpha - 3$ & $\beta^2 = 5\beta - 3$

$x^2 - 5x + 3 = 0$ & $\alpha \neq \beta$

$\alpha + \beta = 5$, $\alpha\beta = 3$

Now

sum of $= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{5^2 - 2.3}{3} = \frac{19}{3}$

Product $= \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$

Q.E. is $x^2 - \frac{19}{3}x + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$

Sol.4 $x^2 + px + q = 0 \begin{matrix}\diagup 2+i\sqrt{3} \\ \diagdown 2+i\sqrt{3}\end{matrix}$

$(-p) = 4 \Rightarrow p = -4$

$q = 4 + 3 \Rightarrow q = 7$

$(p, q) \Rightarrow (-4, 7)$

Sol.5 $(\ell - m)x^2 + \ell x + 1 = 0 \begin{matrix}\diagup \alpha \\ \diagdown 2\alpha\end{matrix}$, $\ell \in R$

$3\alpha = \frac{-\ell}{\ell - m} \Rightarrow \alpha = \frac{-\ell}{3(\ell - m)}$

$(\ell - m)\frac{\ell^2}{9(\ell-m)^2} + \frac{-\ell^2}{3(\ell-m)} + 1 = 0$

$\Rightarrow \ell^2 - 3\ell^2 + 9(\ell - m) = 0$

$\Rightarrow 2\ell^2 - 9\ell + 9m = 0$ Quadratic in ℓ, $\ell \in R$

$\therefore D \geqslant 0 \Rightarrow 81 - 72m \geqslant 0$

$9 - 8m \geqslant 0 \Rightarrow m \leqslant \frac{9}{8}$

Sol.6 $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$

one root is $(-1 + i)$

$\Rightarrow$ second root will be $(-1 - i)$

let other roots α, β

sum $= -4$, product $= -2$

$\Sigma\alpha = \alpha + \beta - 2 = -4$

$\Rightarrow \alpha + \beta = -2$

$\prod\alpha = \alpha\beta(1^2 + 1^2) = -2$

$\Rightarrow \alpha\beta = -1$

Remaining factor of given equation

$x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$

Sol.7 $x^2 - 2cx + ab = 0$ & $D > 0$

$\Rightarrow 4c^2 - 4ab > 0 \Rightarrow c^2 - ab > 0$

$x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$

$D < 0$

$4(a + b)^2 - 4(a^2 + b^2 + 2c^2) < 0$

$\Rightarrow (a + b)^2 - (a^2 + b^2 + 2c^2) < 0$

$\Rightarrow 2ab - 2c^2 < 0$

$\Rightarrow$ $ab - c^2 < 0$ is true $\because$ $c^2 - ac > 0$

Sol.8 $(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$

$D = 0 \Rightarrow 4(b^2 - ac)^2 - 4(a^2 - bc)(c^2 - ab) = 0$

$\Rightarrow b^4 + a^3b + bc^3 - 3ab^2c = 0$

$\Rightarrow b(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow b = 0$ or $a^3 + b^3 + c^3 = 3abc$

Sol.9 $f(x) = 2x^3 + 2x^2 - 7x + 72,$

$x = \frac{3+5i}{2}$

$2x - 3 = 5i$

$4x^2 - 12x + 9 = -25$

$4x^2 - 12x + 34 = 0$

$2x^2 - 6x + 17 = 0$

$f(x) = (2x^2 - 6x + 17)(x + 4) + 4$

$f\left(\frac{3+5i}{2}\right) = 0 + 4 = 4$

Sol.10 If common root is α

$a\alpha^2 + b\alpha + c = 0$

$b\alpha^2 + c\alpha + a = 0$

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

$$\alpha = \frac{ab - c^2}{bc - a^2} = \frac{bc - a^2}{ac - b^2}$$

$(bc - a^2)^2 = (ab - c^2)(ac - b^2)$

$b^2c^2 + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + b^2c^2$

$\Rightarrow a(a^3 + b^3 + c^3) = 3a^2bc \Rightarrow \frac{a^3 + b^3 + c^3}{abc} = 3$

Aliter

By observation common root is 1

(same value occur at common roots)

$\Rightarrow a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc \Rightarrow \frac{a^3 + b^3 + c^3}{abc} = 3$

Sol.11 $ax^2 + bx + c = 0$ &

& $c_1x^2 + b_1x + a_1 = 0$ have a common root α

$a\alpha^2 + b\alpha + c = 0$

$c_1\alpha^2 + b_1\alpha + a_1 = 0$

$$\frac{\alpha^2}{a_1b - b_1c} = \frac{\alpha}{c_1c - a_1a} = \frac{1}{ab_1 - bc_1}$$

$\Rightarrow (cc_1 - aa_1)^2 = (a_1b - b_1c)(ab_1 - bc_1)$

Sol.12.(i) $x^2(y - 1) - x + (y - 1) = 0$

$D \geq 0 \Rightarrow 1 - 4(y - 1)^2 \geq 0$

$\Rightarrow (1 - 2(y - 1))(1 + 2(y - 1)) \geq 0$

$\Rightarrow (2y - 1)(2y - 3) \leq 0 \Rightarrow y \in \left[\frac{1}{2}, \frac{3}{2}\right]$

(ii) $(y - 1)x^2 + 2x(y + 1) + 9(y - 1) = 0$

$D \geq 0 \Rightarrow 4(y + 1)^2 - 36(y - 1)^2 \geq 0$

$\Rightarrow (y + 1)^2 - (3y - 3)^2 \geq 0$

$\Rightarrow (2y - 1)(y - 2) \leq 0 \Rightarrow y \in \left[\frac{1}{2}, 2\right]$

Sol.13 (i) $x^2 - 7x + 10 > 0$

$(x - 5)(x - 2) > 0$

$x \in (-\infty, 2) \cup (5, \infty)$

(ii) $x^2 - 4x + 3 < 0$

$(x - 3)(x - 1) < 0 \Rightarrow x \in (1, 3)$

(iii) $\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x + 1}$

$$\Rightarrow \frac{2x^2 + 2x - 2x^2 - 5x - 2}{(x+2)(2x+1)(x+1)} > 0$$

$$\Rightarrow \frac{3x + 2}{(x+2)(2x+1)(x+1)} < 0$$

$x \in (-2, -1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$

(iv) $\frac{x - 2}{x + 2} > \frac{2x - 3}{4x - 1}$

$$\Rightarrow \frac{(4x^2 - 9x + 2) - (2x^2 + x - 6)}{(x+2)(4x-1)} > 0$$

$$\Rightarrow \frac{2x^2 - 10x + 8}{(x+2)(4x-1)} > 0 \Rightarrow \frac{2(x-1)(x-4)}{(x+2)(4x-1)} > 0$$

$\Rightarrow x \in (-\infty, -2) \cup \left(\frac{1}{4}, 1\right) \cup (4, \infty)$

Sol.14 (i) $(x - 1)^2(x + 1)^3(x - 4) \geq 0$

$x \in (-\infty, -1] \cup [4, \infty) \cup \{1\}$

(ii) $\frac{x^4(x+1)^2(x-2)}{(x-3)^3(x+4)} \geq 0 \Rightarrow x \in (-4, 2] \cup (3, \infty) - \{-1, 0\}$

(iii) $(x^2 - x - 1)(x^2 - x - 7) < -5$

Let $x^2 - x = t$

$\Rightarrow (t - 1)(t - 7) + 5 < 0 \Rightarrow t^2 - 8t + 12 < 0$

$\Rightarrow$ $(t-6)(t-2) < 0$
$\Rightarrow$ $(x^2-x-6)(x^2-x-2) < 0$
$\Rightarrow$ $(x-3)(x+2)(x-2)(x+1) < 0$
$\Rightarrow$ $x \in (-2,-1) \cup (2,3)$

(iv) $\dfrac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

D of denominator is < 0 & a < 0 for all $x \in R$
$\Rightarrow$ $-x^2+3x-4 < 0$ $\Rightarrow$ $(x+2)(x^2-2x+1) \leq 0$
$\Rightarrow$ $(x+2)(x-1)^2 \leq 0$ $\Rightarrow$ $x \in (-\infty, -2] \cup \{1\}$

Sol.15 $f(x) = x^2 - (K+1)x + K^2 + K - 8$
$f(2) < 0$
$\Rightarrow 4 - 2(K+1) + K^2 + K - 8 < 0 \Rightarrow K^2 - K - 6 < 0$
$\Rightarrow$ $(K+2)(K-3) < 0$ $\Rightarrow K \in (-2, 3)$

Sol.16 $(a^2-a+2)x^2 + 2(a-3)x + 9(a^4-16) = 0$
leading coeff. = $a^2-a+2 > 0, \ \forall \ a \in R$
now
$f(0) < 0$
$\Rightarrow$ $9(a^4-16) < 0 \Rightarrow (a^2+4)(a+2)(a-2) < 0$
$\Rightarrow$ $(a+2)(a-2) < 0$ $\Rightarrow$ $a \in (-2, 2)$

Sol.17 $f(x) = x^2 - 2ax + a^2 - 1$
$f(2)\,f(4) < 0$
$(4-4a+a^2-1)(16-8a+a^2-1) < 0$
$(a^2-4a+3)(a^2-8a+15) < 0$
$(a-3)(a-1)(a-5)(a-3) < 0$
$\Rightarrow$ $(a-1)(a-3)^2(a-5) < 0 \Rightarrow a \in (1,5) - \{3\}$

Sol.18 $x^3 + px^2 + qx + r = 0$ & $\alpha\beta + 1 = 0$
$\alpha\beta\eta = -r$
$(-1)\eta = -r \Rightarrow \eta = r$
satisfying equation
$r^3 + pr^2 + qr + r = 0$
$r(r^2 + pr + q + 1) = 0$
$\Rightarrow$ $r = 0$ or $r^2 - pr + q + 1 = 0$

Sol.19 $x^3 + px^2 + qx + r = 0$
$\alpha\beta\gamma = -r$

$$\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$$

$$= \frac{(\alpha\beta\gamma-1)^3}{(\alpha\beta\gamma)^2} = \frac{(-r-1)^3}{r^2} = \frac{-(r+1)^3}{r^2}$$

Sol.20 $2x^3 + x^2 - 7 = 0$

$\Sigma\alpha = -\dfrac{1}{2}$ $\Sigma\alpha\beta = 0,\ \alpha\beta\gamma = \dfrac{7}{2}$

$$\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \sum\frac{\alpha^2+\beta^2}{\alpha\beta} = \sum\frac{\alpha^2+\beta^2+\gamma^2-\gamma^2}{\alpha\beta}$$

$$= \sum\left[\frac{(\alpha^2+\beta^2+\gamma^2)}{\alpha\beta} - \frac{\gamma^2}{\alpha\beta}\right]$$

$$= (\alpha^2+\beta^2+\gamma^2)\left[\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}\right] - \left[\frac{\gamma^2}{\alpha\beta} + \frac{\alpha^2}{\beta\gamma} + \frac{\beta^2}{\alpha\gamma}\right]$$

$$= \left[(\Sigma\alpha)^2 - 2\Sigma\alpha\beta\right]\frac{\Sigma\alpha}{\alpha\beta\gamma} - \frac{\Sigma\alpha^3}{\alpha\beta\gamma}$$

$$= (\Sigma\alpha)^2\frac{(\Sigma\alpha)}{\alpha\beta\gamma} - \frac{(\Sigma\alpha)[(\Sigma\alpha)^2 - 2\Sigma\alpha\beta - \Sigma\alpha\beta] + 3\alpha\beta\gamma}{\alpha\beta\gamma}$$

$$= \frac{(\Sigma\alpha)^3}{\alpha\beta\gamma} - \frac{(\Sigma\alpha)^3}{\alpha\beta\gamma} - \frac{3\alpha\beta\gamma}{\alpha\beta\gamma} = -3$$

Sol.21 $\alpha + \beta = -3$ & $\alpha\beta = \dfrac{a}{2}$

$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} < 2$ $\Rightarrow$ $\dfrac{\alpha^2+\beta^2}{\alpha\beta} < 2$

$\Rightarrow$ $\dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} < 2$ $\Rightarrow$ $\dfrac{9-a}{a/2} < 2$

$\Rightarrow$ $\dfrac{(2a-9)}{a} > 0$ $\Rightarrow$ $a \in (-\infty, 0) \cup \left(\dfrac{9}{2}, \infty\right)$

Sol.22 $\left|\dfrac{x^2+kx+1}{x^2+x+1}\right| < 2 \quad \forall\ x \in R$

$\Rightarrow$ $-2 < \dfrac{x^2+kx+1}{x^2+x+1} < 2$

(i) $-2x^2 - 2x - 2 < x^2 + kx + 1$
$\Rightarrow$ $3x^2 + (k+2)x + 3 > 0$
$D < 0$ $\forall\ x \in R$
$\Rightarrow$ $(k+2)^2 - 36 < 0$ $\Rightarrow$ $(k+8)(k-4) < 0$
$\Rightarrow$ $k \in (-8, 4)$

& **(ii)** $x^2 + kx + 1 < 2x^2 + 2x + 2$
$x^2 + (2-k)x + 1 > 0$
$D < 0$ $\forall\ x \in R$
$\Rightarrow$ $(2-k)^2 - 4 < 0$ $\Rightarrow$ $k(k-4) < 0$
$\Rightarrow$ $k \in (0, 4)$ finally $k \in (0, 4)$

Sol.23 $xy + 3y^2 - x + 4y - 7 = 0$ .... (i)
$2xy + y^2 - 2x - 2y + 1 = 0$ .... (ii)
2(i) – (ii)

$5y^2 + 10y - 15 = 0$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0 \quad \Rightarrow y = 1$ or $y = -3$

If **y = 1,** $x + 3 - x + 4 - 7 \Rightarrow x \in R$

If **y = –3,** $-3x + 27 - x - 12 - 7 = 0$

$-4x + 8 = 0 \quad \Rightarrow x = 2$

$x \in R,\ y = 1$

$x = 2,\ y = -3$

Sol.24 $x^2 + ax + 12 = 0 \quad \to \alpha, \beta$

$x^2 + bx + 15 = 0 \quad \to \alpha, \gamma$

$x^2 + (a + b)x + 36 = 0 \quad \to \alpha, \delta$

Let common root is α

$\alpha^2 + a\alpha + 12 = 0$ ....(i)

$\alpha^2 + b\alpha + 5 = 0$ ....(ii)

$\alpha^2 + (a + b)\alpha + 36 = 0$ ....(iii)

(i) + (ii) – (iii) $\Rightarrow \alpha^2 = 9,\ \alpha \neq -3 \ \Rightarrow \alpha = 3$

$\alpha = 3$ satisfy (i) & (ii)

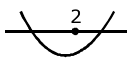
$9 + 3a + 12 \quad \Rightarrow a = -7$

& $9 + 3b + 15 = 0 \Rightarrow b = -8$

$\alpha.\beta = 12 \qquad \alpha.\gamma = 15 \qquad \alpha.\delta = 36$

$\alpha = 3,\ \beta = 4 \qquad \alpha = 3,\ \gamma = 5 \qquad \alpha = 3,\ \delta = 12$

Sol.25 $x^2 - (a - 3)x + a = 0$

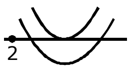


(i) $f(2) < 0$

$\Rightarrow 4 - 2(a - 3) + a < 0$

$\Rightarrow -a + 10 < 0 \ \Rightarrow a \in (10, \infty)$ ...(i)

& (ii) $D \geq 0$



$\Rightarrow (a - 3)^2 - 4a \geq 0$

$\Rightarrow a^2 - 10a + 9 \geq 0$

$\Rightarrow (a - 9)(a - 1) \geq 0 \quad \Rightarrow a \in (-\infty, 1] \cup [9, \infty)$

& $f(2) > 0 \quad \Rightarrow a < 10$

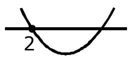
& $-\dfrac{b}{2a} > 2 \quad \Rightarrow \dfrac{a-3}{2} > 2 \quad \Rightarrow a > 7$

$a \in [9, 10)$ ... (ii)

& (iii) $D > 0 \ \Rightarrow a \in (-\infty, 1) \cup (9, \infty)$

& $f(2) = 0 \ \Rightarrow a = 10$

& $-\dfrac{b}{2a} > 2 \ \Rightarrow a > 7$



$\Rightarrow a = 10$ ...(iii)

finally (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow a \in [9, \infty)$

Sol.26 $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$

Let $x^2 + x = t$

$D_1 = (1 + 4t)$

$t > -\dfrac{1}{4} \ \Rightarrow$ real x

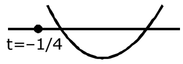
$t < -\dfrac{1}{4} \ \Rightarrow$ non real x

$f(t) = t^2 - at + 4 = 0$

$D = a^2 - 16$

(i) all four real & distinct roots

D > 0 $\Rightarrow a^2 - 16 > 0$



$a \in (-\infty, -4) \cup (4, \infty)$

& $\mathbf{f\left(-\frac{1}{4}\right) > 0} \Rightarrow \dfrac{1}{16} - \dfrac{a}{4} + 4 > 0$

$\Rightarrow 1 - 4a + 64 > 0 \ \Rightarrow a < \dfrac{65}{4}$

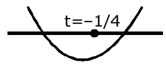
$\mathbf{\dfrac{-b}{2a} > \dfrac{-1}{4}} \ \Rightarrow \dfrac{-a}{2} > \dfrac{-1}{4} \ \Rightarrow a < \dfrac{1}{2}$

$\therefore \ a \in (-\infty, -4)$

(ii) Only two roots are real and distinct

D > 0 $\Rightarrow a\ (-\infty, -4) \cup (4, \infty)$

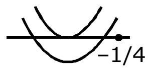
$\mathbf{f\left(-\frac{1}{4}\right) < 0} \ \Rightarrow a > \dfrac{65}{4}$



$\therefore \ a \in \left(\dfrac{65}{4}, \infty\right)$

(iii) All four roots are imaginary

$\mathbf{f\left(-\frac{1}{4}\right) > 0} \ \Rightarrow a < \dfrac{65}{4}$

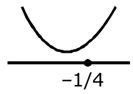


$\mathbf{-\dfrac{b}{2a} < \dfrac{-1}{4}} \ \Rightarrow a > \dfrac{1}{2}$

$\therefore \ a \in \left[4, \dfrac{65}{4}\right)$

or

D < 0 $\Rightarrow a \in (-4, 4)$

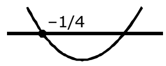


finally $a \in \left(-4, \dfrac{65}{4}\right)$

(iv) Four real roots in which only two are equal

D > 0

$\Rightarrow a \in (-\infty, -4) \cup (4, \infty)$



$\mathbf{f\left(\frac{-1}{4}\right) = 0} \ \Rightarrow a = \dfrac{65}{4}$

$\mathbf{\dfrac{-b}{2a} > \dfrac{-1}{4}} \ \Rightarrow a < \dfrac{1}{2}$

finally $a \in \phi$

Sol.27 $a^2x^2 + (b^2 + a^2 - c^2)x + b^2 = 0, \quad a, b, c \in R^+$

$D < 0$

$(b^2 + a^2 - c^2)^2 - 4a^2b^2 < 0$

$(b^2 + a^2 - c^2 + 2ab)(b^2 + a^2 - c^2 - 2ab) < 0$

$((a + b)^2 - c^2)((a - b)^2 - c^2) < 0$

If $(a+b)^2 - c^2 < 0 \Rightarrow (a-b)^2 - c^2 < 0$

$\therefore\ (a+b)^2 - c^2 \not< 0$

$\Rightarrow (a+b)^2 - c^2 > 0 \quad \& \quad (a-b)^2 - c^2 < 0$

$\Rightarrow (a+b)^2 > c^2 \quad \& \quad (a-b)^2 < c^2$

$\Rightarrow a + b > c \quad \& \quad |a-b| < c$

Sol.28 $\dfrac{a}{(x-a)} + \dfrac{b}{(x-b)} = m$

$a(x-b) + b(x-a) = m(x-a)(x-b)$

$\Rightarrow x(a+b) - 2ab = mx^2 - m(a+b)x + mab$

$\Rightarrow mx^2 - (m+1)(a+b)x + (m+2)ab = 0$

sum of roots = 0

$(m+1)(a+b) = 0$

$m = -1$ or $a + b = 0$

& product of roots < 0 $\Rightarrow \dfrac{(m+2)ab}{m} < 0$

If $m = -1$

$\dfrac{(-1+2)}{-1} ab < 0 \Rightarrow ab > 0$

If $(a+b) = 0 \Rightarrow ab < 0$

$\Rightarrow \dfrac{m+2}{m} > 0$

$m \in (-\infty, -2) \cup (0, \infty)$

Sol.29 $x^3 + qx + r = 0$

$\Sigma x_1 = 0$

$2\alpha + x_3 = 0 \Rightarrow x_3 = -2\alpha$ satisfy equation

$(-2\alpha)^3 + q(-2\alpha) + r = 0$

$(2\alpha)^3 + q(2\alpha) - r = 0 \Rightarrow x^3 + qx - r = 0$

Sol.30 $f(x) = ax^2 + (a-2)x - 2$

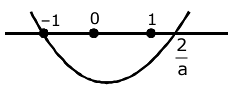
only for two integral values $\Rightarrow f(x) < 0$

$a > 0$

$ax^2 + (a-2)x - 2 = 0$

$$x = \frac{-(a-2) \pm \sqrt{(a-2)^2 + 4a}}{2a} = -\frac{-(a-2) \pm (a+2)}{2a}$$

$x = -1, \dfrac{2}{a}$ $\left(\because \dfrac{2}{a} > 0\right)$



$1 < \dfrac{2}{a} \le 2 \Rightarrow 1 > \dfrac{a}{2} \ge \dfrac{1}{2} \Rightarrow 1 \le a < 2$

Sol.31 $kx^2 + (1-k)x + 5 = 0$

$\alpha + \beta = \dfrac{k-1}{k}, \quad \alpha\beta = \dfrac{5}{k}$

$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{4}{5} \Rightarrow \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \dfrac{4}{5}$

$\Rightarrow \dfrac{\dfrac{(k-1)^2}{k^2} - \dfrac{10}{k}}{5/k} = \dfrac{4}{5} \Rightarrow \dfrac{(k-1)^2 - 10}{(5/k)} = \dfrac{4}{5}$

$\Rightarrow k^2 - 16k + 1 = 0$

$\dfrac{k_1}{k_2} + \dfrac{k_2}{k_1} = \dfrac{(k_1+k_2)^2 - 2k_1k_2}{2k_1k_2} = \dfrac{(16)^2 - 2}{1} = 254$

Sol.32 Let common root is α

$\alpha^2 + b\alpha + c = 0$ .....(i)

$b\alpha^2 + c\alpha + 1 = 0$ .....(ii)

$b(i) - (ii) \Rightarrow (b^2 - c)\alpha + bc - 1 = 0$

$\Rightarrow \alpha = \left(\dfrac{1-bc}{b^2-c}\right)$ put in (i)

$\Rightarrow (1-bc)^2 + (b - b^2c)(b^2 - c) + c(b^2 - c)^2 = 0$

$\Rightarrow 1 + b^2c^2 - 2bc + b^3 + b^2c^2 - b^4c - bc + cb^4 + c^3 - 2b^2c^2 = 0$

$\Rightarrow 1^3 + b^3 + c^3 - 3bc = 0$

$\Rightarrow (1 + b + c)(1 + b^2 + c^2 - bc - b - c) = 0$

EXERCISE – IV HINTS & SOLUTIONS

Sol.1 $ax^2 + bx + c = 0 \Rightarrow \alpha + \alpha^2 = \frac{-b}{a}$, $\alpha^3 = \frac{c}{a}$

$(\alpha + \alpha^2)^3 = \frac{-b^3}{a^3}$

$\Rightarrow \alpha^3 + (\alpha^3)^2 + 3\alpha^3(\alpha + \alpha^2) = \frac{-b^3}{a^3}$

$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + \frac{3c}{a}\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3}$

$\Rightarrow a^2c + ac^2 + b^3 - 3abc = 0$

Sol.2 $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 14}{x^2 - 8x + 32} < 0 \quad \forall x \in R$

Denominator $x^2 - 8x + 32 > 0$

$\Rightarrow ax^2 + 2(a+1)x + 9a + 4 < 0$

$\Rightarrow a < 0$

& $D < 0$

$\Rightarrow 4(a+1)^2 - 4a(9a+4) < 0$

$\Rightarrow a^2 + 2a + 1 - 9a^2 - 4a < 0$

$\Rightarrow 8a^2 + 2a - 1 > 0$

$\Rightarrow (2a+1)(4a-1) > 0$

$\Rightarrow a \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{4}, \infty\right)$

$\therefore a \in \left(-\infty, -\frac{1}{2}\right)$

Sol.3 $\alpha + \beta = \text{cosec } 10^\circ - \sqrt{3}\sec 10^\circ$

$= \frac{2}{2}\frac{[\cos 10^\circ - \sqrt{3}\sin 10^\circ]}{\frac{1}{2}2\sin 10^\circ \cos 10^\circ} = \frac{4\cos 70^\circ}{\sin 20^\circ} = 4$

$\alpha\beta = \frac{1}{2}\text{cosec } 10^\circ - 2\sin 70^\circ$

$= \frac{1 - 2\cdot(2\sin 70^\circ \sin 10^\circ)}{2\sin 10^\circ}$

$= \frac{1 - 2(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ} = \frac{2\cos 80^\circ}{2\sin 10^\circ} = 1$

Q.E. is $x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$\alpha = 2 + \sqrt{3} = \tan 75^\circ = \tan\frac{5\pi}{12}$

$\beta = 2 - \sqrt{3} = \tan 15^\circ = \tan\frac{\pi}{12}$

Sol.4 $y = \frac{6x^2 - 22x + 21}{5x^2 - 8x + 17} \quad \because 5x^2 - 8x + 17 > 0$

$\Rightarrow (5y - 6)x^2 - (18y - 22)x + (17y - 21) = 0$

$\forall x \in R \quad D \geqslant 0 \quad$ if $5y \neq 6$

$(18y - 22)^2 - 4(5y - 6)(17y - 21) \geqslant 0$

$\Rightarrow (9y - 11)^2 - (5y - 6)(17y - 21) \geqslant 0$

$\Rightarrow 81y^2 + 121 - 198y - 85y^2 + 207y - 126 \geqslant 0$

$\Rightarrow 4y^2 - 9y + 5 \leqslant 0 \quad \Rightarrow (y-1)(4y-5) \leqslant 0$

$\Rightarrow y \in \left[1, \frac{5}{4}\right] \Rightarrow y_{min} = 1$

If $y = \frac{6}{5} \Rightarrow x = \frac{3}{2} \Rightarrow y_{min} = 1$

Sol.5 $y = (2p^2 + 1)x^2 + 2x(4p^2 - 1) + 4(2p^2 + 1)$

$y = 2p^2(x+2)^2 + (x-1)^2 + 3$

$y_{min} = 3$ at $x = 1$, & $p = 0$

Sol.6 $ax^2 + bx + c = 0$

If $f(\alpha, \beta) = f(\beta, \alpha) \quad \Rightarrow$ f is symmetric

(i) $f(\alpha, \beta) = \alpha^2 - \beta \neq \beta^2 - \alpha \Rightarrow \alpha \neq \beta$

(ii) $f(\alpha, \beta) = \alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha + \beta) = \beta\alpha(\beta + \alpha)$ symmetric

(iii) $f(\alpha, \beta) = \ln\left(\frac{\alpha}{\beta}\right) \neq \ln\left(\frac{\beta}{\alpha}\right)$

(iv) $f(\alpha, \beta) = \cos(\alpha - \beta)$

$= \cos(\beta - \alpha)$ symmetric

Sol.7 $\frac{1}{x} + \frac{1}{x-a} + \frac{1}{x+b} = 0$; $a, b \in R^+$

$3x^2 - 2(a-b)x - 2ab = 0$

(i) $f\left(\frac{a}{3}\right) f\left(\frac{2a}{3}\right) < 0$

$$\left(\frac{a^2}{3}-\frac{2a(a-b)}{3}-ab\right).\left(\frac{4}{3}a^2-\frac{4a(a-b)}{3}-ab\right)<0$$

$$=-\frac{1}{3}(a^2+ab)\left(\frac{ab}{3}\right)<0$$

a (a + b) (ab) > 0 True

(ii) $f\left(-\frac{2b}{3}\right)f\left(\frac{-b}{3}\right)<0$

$$\left(\frac{4b^2}{3}+\frac{2(a-b)2b}{3}-ab\right)\left(\frac{b^2}{3}+\frac{2b}{3}(a-b)-ab\right)<0$$

$$-\left(\frac{ab}{3}\right)\left(\frac{ab+b^2}{3}\right)<0 \Rightarrow a^2b(a+b)>0 \text{ True}$$

Sol.8 $x^2 - ax + b = 0$

$|\alpha-\beta| < c$ & $D > 0$

$(\alpha-\beta)^2 < c^2$ & $a^2 - 4b > 0$

$(\alpha+\beta)^2 - 4\alpha\beta < c^2$

$a^2 - 4b < c^2 \Rightarrow b > \frac{1}{4}(a^2 - c^2)$

Sol.9 $f(x) = (a-2)x^2 + 2ax + a + 3$ $\alpha, \beta \in (-2, 1)$

Case - I **a = 2** $\Rightarrow 4x + 5 = 0$

$$\Rightarrow x = -\frac{5}{4} \in (-2, 1)$$

Case - II

$$g(x) = x^2+\frac{2a}{(a-2)}x+\frac{(a+3)}{(a-2)}=0$$

(i) D $\geqslant$ 0

$$\Rightarrow \frac{4a^2}{(a-2)^2}-\frac{4(a+3)}{(a-2)}\geqslant 0$$

$\Rightarrow 4a^2 - 4(a+3)(a-2) \geqslant 0$

$\Rightarrow a^2 - a^2 - a + 6 \geqslant 0$ $\Rightarrow a \leqslant 6$

& (ii) g(–2) > 0 $\Rightarrow \frac{a-5}{a-2} > 0$

$\Rightarrow a > 5$ & $a < 2$

& (iii) g(1) > 0 $\Rightarrow \frac{(4a+1)}{(a-2)} > 0$

$\Rightarrow a > 2$ & $a < -\frac{1}{4}$

& (iv) $-2 < \frac{-b}{2a} < 1$

$$-2 < \frac{-2a}{2(a-2)} < 1$$

$\frac{a}{a-2} - 2 < 0$ & $\frac{a}{a-2} + 1 > 0$

$\frac{a-4}{a-2} > 0$ & $\frac{2(a-1)}{(a-2)} > 0$

$a \in (-\infty, 2) \cup (4, \infty)$ & $a \in (-\infty, 1) \cup (2, \infty)$

$\Rightarrow a \in (-\infty, 1) \cup (4, \infty)$

$\therefore a \in \left(-\infty, -\frac{1}{4}\right) \cup (5, 6]$

(case I) $\cup$ (case II)

$$a \in \left(-\infty, -\frac{1}{4}\right) \cup \{2\} \cup (5, 6]$$

Sol.10 Let common root is α

$\alpha^2 + b\alpha + ca = 0$

$\alpha^2 + c\alpha + ab = 0$ $\Rightarrow \alpha = a$

$\alpha\beta = ca \Rightarrow \beta = c$

$\alpha\gamma = ab \Rightarrow \gamma = b$

$\beta + \gamma = b + c$, $\beta\gamma = bc$

Q.E. $x^2 - (b + c)x + bc = 0$

a satisfy the given equations

$a(a + b + c) = 0$

$a \neq 0 \therefore a + b + c = 0 \Rightarrow -(b + c) = a$

$\therefore x^2 + ax + bc = 0$

Sol.11 $\sqrt{x-\frac{1}{x}}+\sqrt{1-\frac{1}{x}} = x$ ...(i)

Rationalise

$$\frac{x-\frac{1}{x}-1+\frac{1}{x}}{\sqrt{x-\frac{1}{x}}-\sqrt{1-\frac{1}{x}}}=x \Rightarrow \sqrt{x-\frac{1}{x}}-\sqrt{1-\frac{1}{x}}=\frac{x-1}{x} \quad \text{... (ii)}$$

$$\text{(i) + (ii)} \Rightarrow \left(\sqrt{x-\frac{1}{x}}-1\right)^2=0$$

$$\Rightarrow x-\frac{1}{x}=1 \Rightarrow x^2-x-1=0 \Rightarrow x=\frac{1\pm\sqrt{5}}{2}$$

$\frac{1-\sqrt{5}}{2}$ reject (doesn't satisfy (i)) $\Rightarrow x = \frac{1+\sqrt{5}}{2}$

Sol.12 $-3 < \left(\frac{x^2+ax-2}{x^2+x-1}\right) < 2$

(i) $-3x^2 - 3x - 3 < x^2 + ax - 2$

$\forall x \in R$

$4x^2 + (a+3)x + 1 > 0$

$D < 0 \Rightarrow (a+3)^2 - 16 < 0$

$\Rightarrow -4 < (a+3) < 4 \Rightarrow a \in (-7, 1)$

& (ii) $x^2 + ax - 2 < 2x^2 + 2x + 2$

$\Rightarrow x^2 + (2-a)x + 4 > 0$

$D < 0 \Rightarrow (2-a)^2 - 16 < 0$

$\Rightarrow -4 < 2 - a < 4 \Rightarrow a \in (-2, 6)$

$\therefore$ finally $a \in (-2, 1)$

Sol.13 $\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)} = \frac{\left(x+\frac{1}{x}\right)^6 - \left(x^3+\frac{1}{x^3}\right)^2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$

$\frac{A^6 - B^2}{A^3 + B} = \frac{A^3 + B}{A^3 + B}\ (A^3 - B)$

$= \left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right) = 3\left(x+\frac{1}{x}\right) = 6$

Sol.14 $x^2 + 18 + 30 = \sqrt{x^2+18x+45}$

Let $\sqrt{x^2+18x+45} = t$, $t > 0$

$t^2 - 15 = 2t$

$t^2 - 2t - 15 = 0$

$(t-5)(t+3) = 0$

$t \neq -3$

$\therefore$ $t = 5$

$x^2 + 18x + 45 = 25$

$x^2 + 18x + 20 = 0$

$D > 0$

$\therefore$ $\alpha\beta = 20$

Sol.15 $x^3 - 3x^2 + 1 = 0$

$\frac{\alpha}{\alpha - 2} = t \Rightarrow \alpha = t\alpha - 2t$

$\Rightarrow \alpha = \frac{2t}{t-1}$ satisfy given eqution

$\left(\frac{2t}{t-1}\right)^3 - 3\left(\frac{2t}{t-1}\right)^2 + 1 = 0$

$8t^3 - 12t^2(t-1) + (t-1)^3 = 0$

$\Rightarrow 8t^3 - 12t^3 + 12t^2 + t^3 - 3t^2 + 3t - 1 = 0$

$\Rightarrow -3t^3 + 9t^2 + 3t - 1 = 0$

$\Rightarrow 3t^3 - 9t^2 - 3t + 1 = 0$

or $3x^3 - 9x^2 - 3x + 1 = 0$

If roots are $(\alpha - 2), (\beta - 2), (r - 2)$

Let $t = \alpha - 2$

$\alpha = t + 2$

$(t+2)^3 - 3(t+2)^2 + 1 = 0$

$\Rightarrow t^3 + 6t^2 + 12t + 8 - 3t^2 - 12t - 12 + 1 = 0$

$\Rightarrow t^3 + 3t^2 - 3 = 0$

$(\alpha - 2)(\beta - 2)(r - 2) = -(-3) = 3$

Sol.16 $x^2 - 2x - a^2 + 1 = 0 \to \alpha, \beta$ ....(i)

$f(x) = x^2 - 2(a+1)x + a(a-1) \to \gamma, \delta$ ....(ii)

from (i)

$(x-1)^2 - a^2 = 0$

$x = (1+a)$, or $x = (1-a)$

f(1 + a) < 0

$\Rightarrow (a+1)^2 - 2(a+1)^2 + a(a-1) < 0$

$\Rightarrow -(a+1)^2 + a(a-1) < 0$

$\Rightarrow -a^2 - 2a - 1 + a^2 - a < 0$

$\Rightarrow 3a + 1 > 0 \Rightarrow a > \frac{-1}{3}$

& f(1 – a) < 0

$\Rightarrow (1-a)^2 - 2(a+1)(1-a) + a(a-1) < 0$

$\Rightarrow 1 + a^2 - 2a + 2a^2 - 2 + a^2 - a < 0$

$\Rightarrow 4a^2 - 3a - 1 < 0 \Rightarrow (4a+1)(a-1) < 0$

$\Rightarrow a \in \left(-\frac{1}{4}, 1\right)$ finally $\mathbf{a \in \left(-\frac{1}{4}, 1\right)}$

Sol.17 $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$

$\alpha\beta\gamma\delta = -1984 \quad \because \alpha\beta = -32$

$\therefore \gamma\delta = \frac{-1984}{-32} \Rightarrow \gamma\delta = 62$

Equation may be written as

$(x^2 + px - 32)(x^2 + qx + 62) = 0$

$\Rightarrow x^4 + (p+q)x^3 + (pq+30)x^2$

$+ (62p - 32q)x - 1984 = 0$

$p + q = -18$ & $pq + 30 = k$

& $62p - 32q = 200$

$31p - 16q = 100$

$31p - 16(-18 - p) = 100$

$47p = -188 \Rightarrow p = -4$ & $q = -14$

$k = pq + 30$

$= (-4)(-14) + 30 \Rightarrow k = 86$

Sol.18 $f(x) = x^3 + px^2 + qx + 72$

Let common root is α

$\left.\begin{array}{l}\alpha^2 + a\alpha + b = 0 \\ \alpha^2 + b\alpha + a = 0\end{array}\right\} \Rightarrow \alpha = 1$

$x^2 + ax + b = 0 \rightarrow 1, \beta \Rightarrow \beta = b$

$x^2 + bx + a = 0 \rightarrow 1, \gamma \Rightarrow \gamma = a$

The roots of given equation

$1, \beta, \gamma$ or $1, a, b$

$-p = 1 + a + b = 0 \Rightarrow a + b = -1$

$\Rightarrow p = 0$

$q = (a + b) + ab$

$q = -1 - 72 \Rightarrow q = -73$

$1^2 + a^2 + b^2 = (1 + a + b)^2 - 2(a + b + ab)$

$= 0^2 - 2(-73) = 146$

Sol.19 $x^2 + 2(k - 1)x + k + 5 = 0$

Case - I

(i) $D \geqslant 0$

$\Rightarrow 4(k - 1)^2 - 4(k + 5) \geqslant 0$

$\Rightarrow k^2 - 3k - 4 \geqslant 0 \Rightarrow (k + 1)(k - 4) \geqslant 0$

$\Rightarrow k \in (-\infty, 1] \cup [4, \infty)$

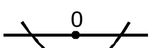
& (ii) $f(0) \geqslant 0 \Rightarrow k + 5 \geqslant 0 \Rightarrow k \in [-5, \infty)$

& (iii) $\frac{-b}{2a} > 0 \Rightarrow \frac{-2(k-1)}{2} > 0$

$\Rightarrow k \in (-\infty, 1) \quad \therefore \quad k \in [-5, -1]$

Case - II $f(0) < 0 \Rightarrow k + 5 < 0$

$\Rightarrow k \in (-\infty, 5)$



Finally $k \in$ (Case - I) $\cup$ (Case - II)

$k \in (-\infty, -1]$

Sol.20 $2\log_{\frac{1}{25}}(bx + 28) = -\log_5(12 - 4x - x^2)$

$\Rightarrow -\log_5(bx + 28) = -\log_5(12 - 4x - x^2)$

$bx + 28 = 12 - 4x - x^2$

$\Rightarrow x^2 + (b + 4)x + 16 = 0 \quad \ldots(i)$

has only one sol. D = 0

$(b + 4)^2 - 64 = 0 \Rightarrow b = -12, 4$

$bx + 28 > 0 \quad \ldots(ii)$

& $12 - 4x - x^2 > 0 \Rightarrow x \in (-6, 2) \quad \ldots(iii)$

put in (i)

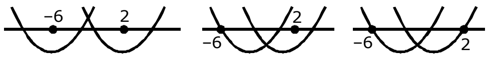
$b = 4 \Rightarrow (x + 4)^2 = 0$

$\Rightarrow x = -4 \in (-6, 2) \Rightarrow b = 4$

$b = -12 \Rightarrow (x - 4)^2 = 0$

$\Rightarrow x = 4 \notin (-6, 2)$, $b = -12$ reject

Now equation (i) has only one root in $x \in (-6, 2)$



In all cases $\Rightarrow f(-6)f(2) \leqslant 0$

$\Rightarrow (36 - 6(4 + b) + 16)(4 + 2(4 + b) + 16) \leqslant 0$

$\Rightarrow (28 - 6b)(28 + 2b) \leqslant 0$

$\Rightarrow (b + 14)(3b - 14) \geqslant 0$

$b \in (-\infty, -14] \cup \left[\frac{14}{3}, \infty\right)$

but –14 doesn't satisfy (i) & (ii)

$\Rightarrow b \in (-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$

Sol.21 $x^2 - ax + 2 = 0$

(i) $D \geqslant 0 \Rightarrow a^2 - 8 \geqslant 0$

$\Rightarrow a \in \left(-\infty, -2\sqrt{2}\right] \cup \left[2\sqrt{2}, \infty\right)$

& (ii) $f(0) > 0 \Rightarrow a \in R$

& (iii) $f(3) > 0 \Rightarrow 9 - 3a + 2 > 0$

$\Rightarrow 3a - 11 < 0 \Rightarrow a \in \left(-\infty, \frac{11}{3}\right)$

& (iv) $0 < \frac{-b}{2a} < 3 \Rightarrow 0 < \frac{a}{2} < 3 \Rightarrow 0 < a < 6$

finally $a \in \left[2\sqrt{2}, \frac{11}{3}\right)$

Sol.22 $1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) \geqslant \log_2(cx^2 + c)$

$\Rightarrow 1 + \log_2\frac{(4x^2 + 4x + 7)}{2} \geqslant \log_2 c(x^2 + 1)$

(i) $4x^2 + 4x + 7 > 0$

$\Rightarrow D < 0 \quad \forall x \in R$

(ii) $c(x^2 + 1) > 0 \Rightarrow c > 0$

$\Rightarrow \log_2(4x^2 + 4x + 7) \geqslant \log_2 c(x^2 + 1)$

$\Rightarrow 4x^2 + 4x + 7 \geqslant c(x^2 + 1)$

$\Rightarrow (4 - c)x^2 + 4x + (7 - c) \geqslant 0$

$D = 16 - 4(4 - c)(7 - c)$

Case - I $c < 4$ for all $x \in R \Rightarrow D \leqslant 0$

$(4 - c)(7 - c) - 4 \geqslant 0$

$c^2 - 11c + 24 \geqslant 0$

$(c - 3)(c - 8) \geqslant 0$

$c \in (-\infty, 3] \cup [8, \infty) \cap c < 4 \Rightarrow c \in (-\infty, 3]\ \forall\ x \in R$

or $D > 0 \Rightarrow c \in (3, 8) \cap c < 4$

$\therefore$ $c \in (-\infty, 4)$ at least one sol.

Case - II $c = 4$

$4x + 3 \geqslant 0$ at least one sol.

$c = 4$

Case - III $c > 4$

& $D \geqslant 0$

$(c - 3)(c - 8) \leqslant 0$

$c \in [3, 8]$ & $c > 4$

$c \in (4, 8]$

$\therefore$ $c \in (-\infty, 4) \cup \{4\} \cup (4, 8] \Rightarrow c \in (-\infty, 8]$

but $c > 0 \Rightarrow c \in (0, 8]$

Sol.23 $(a^2 - 6a + 5)x^2 - \sqrt{a^2 + 2a}\, x + (6a - a^2 - 8) = 0$

Q.E. $\Rightarrow a^2 - 6a + 5 \neq 0$

$$\therefore \quad x^2 - \frac{\sqrt{a^2+2a}}{a^2-6a+5}x - \frac{(a^2-6a+8)}{(a^2-6a+5)} = 0$$

$$f(0) < 0 \Rightarrow \frac{-(a^2-6a+8)}{(a^2-6a+5)} < 0 \Rightarrow \frac{(a-4)(a-2)}{(a-5)(a-1)} > 0$$

$a \in (-\infty, 1) \cup (2, 4) \cup [5, \infty)$

& $a^2 + 2a \geqslant 0 \Rightarrow a(a + 2) \geqslant 0$

$a \in (-\infty, -2] \cup [0, \infty)$

finally $a \in (-\infty, -2] \cup [0, 1) \cup (2, 4) \cup (5, \infty)$

Sol.24 $g(x) = x^3 + px^2 + qx + r$ where $p, q, r \in I$

$g(0) = r = \text{odd}$

$g(-1) = -1 + p - q + r = \text{odd}$

$\Rightarrow p - q - 1 = \text{even}$

$\Rightarrow p - q = \text{odd}$

Let

α, β, γ are Integral roots

$\alpha\beta\gamma = -r = \text{odd} \Rightarrow \alpha, \beta, \gamma$ are odd

$\alpha + \beta + \gamma = -p = \text{odd} \Rightarrow (\because \alpha, \beta, \gamma$ are odd)

but q is odd

$\because$ $\alpha\beta, \beta\gamma, \gamma\alpha$ are odd $\Rightarrow p - q$ is not odd

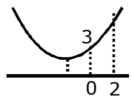
which is constraduction $\Rightarrow \alpha, \beta, \gamma$ are not integral

Sol.25 $f(x) = 4x^2 - 4px + (p^2 - 2p + 2)$ $\quad 0 \leqslant x \leqslant 2$

$$\frac{-b}{2a} = \frac{-(-4p)}{2\cdot 4} = \frac{p}{2}$$

Case - I $\frac{p}{2} < 0 \quad p < 0$

least value of at 0, $f(0) = 3$



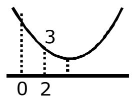
$p^2 - 2p + 2 = 3$

$p^2 - 2p - 1 = 0$

$p = \frac{2 \pm 2\sqrt{2}}{2} \Rightarrow p = 1 - \sqrt{2} \quad \because \quad p < 0$

Case - II $\frac{p}{2} > 2 \Rightarrow p > 4$

least value at 2



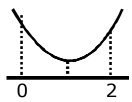
$f(2) = 3$

$p^2 - 10p + 18 = 3$

$p^2 - 10p + 15 = 0$

$p = \frac{10 \pm \sqrt{40}}{2} \Rightarrow p = 5 + \sqrt{10} \quad \because \quad p > 4$

Case - III $0 < \frac{p}{2} < 2$



$\Rightarrow 0 < p < 4$

least value $= \frac{-D}{4a} = 3 \Rightarrow -2(p - 1) = 3$

$p = -1/2$ reject $\because$ $p \in (0, 4)$

$\therefore$ $p = \left(1 - \sqrt{2}\right)$ or $(5 + \sqrt{10})$

Sol.26 $P(x) = x^2 + bx + c$ is a factor

$x^4 + 6x^2 + 25 = x^4 + 5^2 + 6x^2$

$= (x^2 + 5)^2 - 4x^2$

$= (x^2 + 2x + 5)(x^2 - 2x + 5)$

Let $(x^2 - 2x + 5)$ is a factor also of

$3x^4 + 4x^2 + 28x + 5$

$= 3x^2(x^2 - 2x + 5) + 6x(x^2 - 2x + 5) + 1(x^2 - 2x + 5)$

$= (x^2 - 2x + 5)(3x^2 + 6x + 1)$

$\therefore$ common factor of both polynomial is

$P(x) = x^2 - 2x + 5$

$P(1) = 1 - 2 + 5 = 4$

EXERCISE – V HINTS & SOLUTIONS

Sol.1 $(x-a)(x-b)+c=0$

$\Rightarrow \quad (x-\alpha)(x-\beta)=0$

Now $\quad (x-\alpha)(x-\beta)=c$

$(x-a)(x-b)+c=c$

$\Rightarrow \quad x=a, b$

Sol.2.(a) $3x^2+px+3=0$

$$\alpha+\alpha^2=\frac{-p}{3}$$

& $\alpha^3=1 \Rightarrow \alpha=1$ or $\alpha^2+\alpha+1=0$

$p=-3(1+1^2)=-6$ $\quad$ {reject $\because$ $p>0$}

If $\alpha^2+\alpha+1=0$

$$\Rightarrow \alpha^2+\alpha=-1=-\frac{p}{3} \quad \Rightarrow \quad p=3$$

(b) $x^2+bx+c=0$

$\alpha\beta<0 \quad \& \quad \alpha+\beta=-b<0$

$\alpha<0 \quad \& \quad \beta>0 \quad (\because \alpha<\beta)$

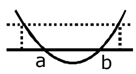
& $|\alpha|>\beta$

(c) $(x-a)(x-b)-1=0, \quad b>a$

roots of $(x-a)(x-b)-1=0$

one is in $(-\infty, a)$

other is in (b, ∞)



(d) $ax^2+bx+c=0 \quad (a\neq 0)$

$Ax^2+Bx+C=0 \quad (A\neq 0)$

$$\alpha+\beta+2\delta=\frac{-B}{A} \quad \Rightarrow \quad \delta=\frac{1}{2}\left(\frac{b}{a}-\frac{B}{A}\right)$$

$$\& \quad \alpha\beta+(\alpha+\beta)\delta+\delta^2=\frac{C}{A}$$

$$\Rightarrow \quad \frac{c}{a}+\left(\frac{-b}{a}\right)\frac{1}{2}\left(\frac{b}{a}-\frac{B}{A}\right)+\frac{1}{4}\left(\frac{b}{a}-\frac{B}{A}\right)^2=\frac{C}{A}$$

$$\Rightarrow \frac{c}{a}-\frac{1}{2}\frac{b^2}{a^2}+\frac{1}{2}\frac{bB}{aA}+\frac{1}{4}\frac{b^2}{a^2}+\frac{1}{4}\frac{B^2}{A^2}-\frac{1}{2}\frac{bB}{aA}=\frac{C}{A}$$

$$\Rightarrow \frac{c}{a}-\frac{1}{4}\frac{b^2}{a^2}=\frac{C}{A}-\frac{1}{4}\frac{B^2}{A^2} \quad \Rightarrow \quad \frac{b^2-4ac}{a^2}=\frac{B^2-4AC}{A^2}$$

Sol.3 $a^3x^2+abcx+c^3=0 \to \alpha',\beta' \quad \ldots$ (i)

$$\alpha'+\beta'=\frac{-abc}{a^3} \quad \& \quad \alpha'\beta'=\left(\frac{c}{a}\right)^3=\alpha^3\beta^3$$

$$=\left(\frac{-b}{a}\right)\left(\frac{c}{a}\right)=(\alpha+\beta)(\alpha\beta)$$

the roots of (i) are $\alpha^2\beta, \alpha\beta^2$

Sol.4 $x^2-|x+2|+x>0$

Case - I $\quad x\geqslant -2 \Rightarrow x^2-x-2+x>0$

$\Rightarrow \quad x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

$\therefore \quad x\in\left[-2,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

Case - II $\quad x<-2$

$\Rightarrow \quad x^2+x+2+x>0$

$\Rightarrow \quad x^2+2x+2>0$

$\Rightarrow \quad x\in R \quad (\because D<0)$

$\therefore \quad x\in(-\infty,-2)$

$x\in$ (Case - I) $\cup$ (Case - II)

$\Rightarrow \quad x\in\left(-\infty,-\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$

Ailter $|x+2|<x^2+x \Rightarrow -(x^2+x)<x+2<x^2+x$

Sol.5 $x^2+(a-b)x+(1-a-b)=0 \quad a, b\in R$

$D>0$

$(a-b)^2-4(1-a-b)>0$

$\Rightarrow a^2+b^2-2ab-4+4a+4b>0$

$b^2+b(4-2a)+(a^2+4a-4)>0$

It's true $\forall$ $b\in R$

$\Rightarrow D<0$

$(4-2a)^2-4(a^2+4a-4)<0$

$\Rightarrow 4-4a+a^2-a^2-4a+4<0$

$\Rightarrow -8a+8<0 \quad \Rightarrow a>1$

Sol.6 **(a)** $x^2+px+q=0\to\alpha,\alpha^2$

$\alpha+\alpha^2=-p, \quad \alpha^3=q$

$(\alpha+\alpha^2)^3=-p^3$

$\alpha^3+(\alpha^3)^2+3\alpha^3(\alpha+\alpha^2)=-p^3$

$q+q^2+3q(-p)=-p^3$

$\Rightarrow p^3+q^2+q(1-3p)=0$

(b) $x^2+2ax+(10-3a)>0 \quad \forall \; x\in R$

$D<0 \quad \Rightarrow 4a^2-4(10-3a)<0$

$\Rightarrow a^2+3a-10<0$

$\Rightarrow (a+5)(a-2)<0 \quad \Rightarrow a\in(-5, 2)$

Sol.7 $$2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1} \quad t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

Let $\quad y = \dfrac{5x^2 - 2x + 1}{3x^2 - 2x - 1}$

$(5 - 3y)x^2 + (2y - 2)x + (y + 1) = 0$

$D \geq 0 \qquad\qquad x \in R$

$4(y - 1)^2 - 4(y + 1)(5 - 3y) \geq 0$

$y^2 - 2y + 1 + 3y^2 - 2y - 5 \geq 0$

$\Rightarrow 4y^2 - 4y - 4 \geq 0 \qquad \Rightarrow y^2 - y - 1 \geq 0$

$$y \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$$

$-2 \leq y = 2\sin t \leq 2$

$$2\sin t \in \left[-2, \frac{1-\sqrt{5}}{2}\right] \cup \left[\frac{1+\sqrt{5}}{2}, 2\right]$$

$$\sin t \in \left[-1, \frac{1-\sqrt{5}}{4}\right] \cup \left[\frac{1+\sqrt{5}}{4}, 1\right]$$

$$t \in \left[\frac{-\pi}{2}, \frac{-\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$$

Sol.8 **(a)** $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$

$D \geq 0$

$4(a + b + c)^2 - 4\cdot 3\lambda\,(ab + bc + ca) \geq 0$

$(a + b + c)^2 - 3\lambda\,(ab + bc + ca) \geq 0$

$a^2 + b^2 + c^2 + (2 - 3\lambda)(ab + bc + ca) \geq 0$

$$(3\lambda - 2) \leq \frac{a^2 + b^2 + c^2}{ab + bc + ca}$$

$$\Rightarrow (3\lambda - 2) < \frac{\Sigma a^2}{\Sigma ab} \quad ......(i)$$

we know $a > c - b$, $b > a - c$, $c > b - a$

$\Rightarrow a^2 + b^2 + c^2 > 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$

$$\Rightarrow 2 > \frac{\Sigma a^2}{\Sigma ab} \quad ....(ii)$$

from (i) & (ii) $(3\lambda - 2) < 2 \Rightarrow \lambda < \dfrac{4}{3}$

(b) $x^2 - 10cx - 11d = 0 \begin{smallmatrix}\nearrow a \\ \searrow b\end{smallmatrix}$

$x^2 - 10ax - 11b = 0 \begin{smallmatrix}\nearrow c \\ \searrow d\end{smallmatrix}$

$$\left.\begin{array}{l} a + b = 10c \;\; .....(i) \;,\; ab = -11d \\ c + d = 10a \;\; .....(ii) \;,\; cd = -11b \end{array}\right\}$$

$(a + b + c + d) = 10(a + c) \quad , \; abcd = 121\,bd$

$(b + d) = 9(a + c) \qquad\qquad ac = 121$

$\& \; (ab + cd) = -11\,(b + d)$

$\Rightarrow$ (i) a + (ii) c

$\Rightarrow a^2 + c^2 + (ab + cd) = 20ac$

$\Rightarrow a^2 + c^2 - 11(b + d) = 20ac$

$a^2 + c^2 - 99(a + c) = 20ac$

$(a + c)^2 - 99(a + c) - 22ac = 0 \qquad ac = 121$

$(a + c)^2 - 99(a + c) - 22(121) = 0 \quad (a + c) = t$(let)

$(t - 121)(t + 22) = 0$

$a + c = 121 \qquad$ a, b, c is positive

$t \neq -22$

$\therefore \; a + b + c + d = 10(121) = 1210$

Sol.9 **(a)** $x^2 - px + r = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}$, $x - qx + r = \begin{smallmatrix}\nearrow \alpha/2 \\ \searrow 2\beta\end{smallmatrix}$

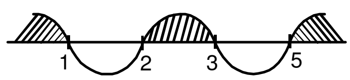
$\alpha + \beta = p$ ...(i), $\dfrac{\alpha}{2} + 2\beta = q$ ...(ii), $\alpha\beta = r$ ...(iii)

from (i) & (ii) $\qquad \beta = \dfrac{(2q - p)}{3} \;\; \& \;\; \alpha = \dfrac{2(2p - q)}{3}$

$r = \dfrac{2}{9}(2p - q)(2q - p)$

(b) $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6} = \dfrac{(x-1)(x-5)}{(x-2)(x-3)} > 0$

$\dfrac{(x-1)(x-5)}{(x-2)(x-3)} > 0$



(R) $f(x) > 0 \Rightarrow (-\infty, 1) \cup (2, 3) \cup (5, \infty)$

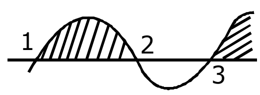
& (Q) $f(x) < 0 \Rightarrow x \in (1, 2) \cup (3, 5)$

(S) $f(x) < 1$

$\dfrac{x^2 - 6x + 5}{x^2 - 5x + 6} - 1 < 0$

$\Rightarrow \dfrac{-x - 1}{x^2 - 5x + 6} < 0 \Rightarrow \dfrac{(x + 1)}{(x-2)(x-3)} > 0$

$x \in (-1, 2) \cup (3, \infty)$



(P) $0 < f(x) < 1$

$\Rightarrow \; R \cap S$

$x \in (-1, 1) \cup (5, \infty)$

(A) $-1 < x < 1$ then $f(x) > 0$ (R)

$f(x) < 1$ (S)

$0 < f(x) < 1$ (P)

(B) $1 < x < 2$ then $f(x) < 0$ (Q)

$f(x) < 1$ (S)

(C) $3 < x < 5$ then $f(x) < 0$ (Q)

$f(x) < 1$ (S)

(D) $x > 5$ then $f(x) > 0$ (R)

$f(x) < 1$ (S)

$0 < f(x) < 1$ (P)

Sol.10 $a, b, c, p, q \in R$

$x^2 + 2px + q = 0 \; \langle \alpha, \beta$ ....(i)

$ax^2 + 2bx + c = 0 \; \langle \alpha, 1/\beta$ ....(ii)

where $\beta^2 \notin \{-1, 0, 1\}$

one common root

state – I from (i) $D \ge 0$

$\Rightarrow 4p^2 - 4q \ge 0$

$\Rightarrow (p^2 - q) \ge 0$ ....(iii)

from (ii) $D \ge 0$

$\Rightarrow 4b^2 - 4ac \ge 0 \quad \Rightarrow (b^2 - ac) \ge 0$ ....(iv)

from (iii) & (iv) $(p^2 - q)(b^2 - ac) \ge 0$ True

state – II one common root is α

$\alpha^2 + 2p\alpha + q = 0$

$a\alpha^2 + 2b\alpha + c = 0$

$$\Rightarrow \alpha = \frac{1}{2}\frac{(c - aq)}{(ap - b)}$$

If both roots are common $\Rightarrow \beta = \pm 1$

But $\beta \ne \pm 1, 0$

$$\frac{1}{a} = \frac{p}{b} \ne \frac{q}{c} \Rightarrow c \ne qa$$

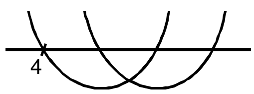
$$\text{or} \quad \frac{1}{a} \ne \frac{p}{b} = \frac{q}{c} \Rightarrow b \ne pq$$

Sol.11 $x^2 - 8kx + 16(k^2 - k + 1) = 0$

(i) $D > 0$

$64k^2 - 64(k^2 - k + 1) > 0$

$\Rightarrow k > 1$



& (ii) $f(4) \ge 0$

$16 - 32k + 16(k^2 - k + 1) \ge 0$

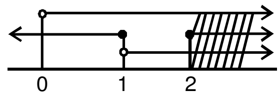
$k^2 - 3k + 2 \ge 0$

$(k - 1)(k - 2) \ge 0 \Rightarrow k \in (-\infty, 1] \cup [2, \infty)$

& (iii) $\frac{-b}{2a} > 0 \Rightarrow \frac{8k}{2} > 0 \Rightarrow k > 0$

$k \in [2, \infty)$

smallest value of k is 2



Sol.12 B

$\alpha + \beta = -p$ & $\alpha^3 + \beta^3 = q$

$(\alpha + \beta)^3 = (-p)^3$

$\Rightarrow (\alpha^3 + \beta^3) + 3\alpha\beta(\alpha + \beta) = -p^3$

$\Rightarrow q + 3\alpha\beta(-p) = -p^3$

$$\Rightarrow \alpha\beta = \frac{p^3 + q}{3p}$$

$$\text{sum of roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{p^3 - 2q}{p^3 + q}$$

$$\text{product of root} = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

$$\text{Q.E. is } x^2 - \left(\frac{p^3 - 2q}{p^3 + q}\right)x + 1 = 0$$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

13. C

$x^2 - 6x - 2 = 0$

$$\frac{\alpha_{10} - 2\alpha_8}{2\alpha_9} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} \quad ....(1)$$

$$\left.\begin{aligned} \alpha^2 - 2 &= 6\alpha \\ \& \quad \beta^2 - 2 &= 6\beta \end{aligned}\right\} \text{put in (1)}$$

14. B

Let common root is α.

$\alpha^2 + b\alpha - 1 = 0$ ....(1)

$\alpha^2 + \alpha + b = 0$ ....(2)

$$\text{on subtracting} \quad \alpha = \frac{b + 1}{b - 1}$$

put in (i) $\Rightarrow b = 0$ or $\pm \sqrt{3}\, i$

Answer Ex–I SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. B	**2.** B	**3.** C	**4.** A	**5.** A	**6.** A	**7.** C
8. A	**9.** B	**10.** B	**11.** B	**12.** A	**13.** C	**14.** C
15. C	**16.** D	**17.** B	**18.** B	**19.** B	**20.** D	**21.** C
22. B	**23.** B	**24.** B	**25.** A	**26.** B	**27.** D	**28.** C
29. C	**30.** D	**31.** A	**32.** B	**33.** A	**34.** A	**35.** B
36. B	**37.** D	**38.** A	**39.** B	**40.** A	**41.** A	**42.** B
43. D	**44.** B	**45.** C	**46.** A	**47.** C	**48.** A	**49.** D
50. B	**51.** B	**52.** B	**53.** D	**54.** C	**55.** D	**56.** B
57. B	**58.** A	**59.** C	**60.** C	**61.** D	**62.** B	**63.** A
64. B	**65.** B					

Answer Ex–II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. BD	**2.** BCD	**3.** ABCD	**4.** AD	**5.** BD	**6.** CD	**7.** AD
8. AD	**9.** ABCD	**10.** AB				

Answer Ex–III SUBJECTIVE QUESTIONS

1. $a = 2$; No real value of x.

2. (i) $acx^2 + b(a + c)x + (a + c)^2 = 0$ (ii) $a^2x^2 + (2ac - 4a^2 - b^2)x + 2b^2 + (c - 2a)^2 = 0$

3. $3x^2 - 19x + 3 = 0$ **4.** $(-4, 7)$ **6.** $-1 \pm \sqrt{2}\,;\ -1 \pm \sqrt{-1}$ **9.** 4 **10.** 3

12. (i) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (ii) $\left[\frac{1}{2}, 2\right]$

13. (i) $x \in (-\infty, 2) \cup (5, \infty)$ (ii) $x \in (1, 3)$

(iii) $x \in (-2, -1) \cup (-2/3, -1/2)$ (iv) $x \in (-\infty, -2) \cup \left(\frac{1}{4}, 1\right) \cup (4, \infty)$

14. (i) $x \in (-\infty, -1] \cup \{1\} \cup [4, \infty)$ (ii) $x \in (-4, -1) \cup (-1, 0) \cup (0, 2) \cup (3, \infty)$

(iii) $x \in (-2, -1) \cup (2, 3)$ (iv) $x \in (-\infty, -2] \cup \{1\}$

15. $K \in (-2, 3)$ **16.** $a \in (-2, 2)$ **17.** $a \in (1, 5) - \{3\}$ **19.** $-\frac{(r+1)^3}{r^2}$ **20.** -3

21. $(-\infty, 0) \cup (9/2, \infty)$ **22.** $k \in (0, 4)$ **23.** $x \in R$ if $y = 1$, $x = 2$ if $y = -3$

24. $a = -7$, $b = -8$; roots (3, 4), (3, 5), (3, 12) **25.** $a \in [9, \infty)$

26. (i) $a \in (-\infty, -4)$ (ii) $a \in \left(\frac{65}{4}, \infty\right)$ (iii) $a \in \left(-4, \frac{65}{4}\right)$ (iv) $a \in \phi$

28. $a + b = 0$, $m \in (-\infty, -2) \cup (0, \infty)$ or $m = -1$, $ab > 0$ **29.** $x^3 + qx - r = 0$ **30.** $[1, 2)$

31. 254

Answer Ex–IV ADVANCED SUBJECTIVE QUESTIONS

2. $a \in \left(-\infty, -\frac{1}{2}\right)$ **3.** $x^2 - 4x + 1 = 0$; $\alpha = \tan\left(\frac{\pi}{12}\right)$; $\beta = \tan\left(\frac{5\pi}{12}\right)$ **4.** 1

5. minimum value 3 when x = 1 and p = 0

6. (a) (ii) and (iv); (b) $x^2 - p(p^4 - 5q^2)\,x + p^2q^2(p^2 - 4q)\,(p^2 - q) = 0$

9. $\left(-\infty, -\frac{1}{4}\right) \cup \{2\} \cup (5, 6]$ **11.** $x = \frac{\sqrt{5}+1}{2}$ **12.** $-2 < a < 1$ **13.** $y_{min} = 6$ **14.** 20

15. $3y^3 - 9y^2 - 3y + 1 = 0$; $(\alpha - 2)(\beta - 2)(\gamma - 2) = 3$ **16.** $a \in \left(-\frac{1}{4}, 1\right)$ **17.** k = 86

18. 146 **19.** $K \le -1$ **20.** $(-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right)$ **21.** $2\sqrt{2} \le a < \frac{11}{3}$

22. (0, 8] **23.** $(-\infty, -2] \cup [0, 1) \cup (2, 4) \cup (5, \infty)$ **25.** $a = 1 - \sqrt{2}$ or $5 + \sqrt{10}$ **26.** P(1) = 4

Answer Ex–V JEE PROBLEMS

1. (a, b) **2.** **(a)** C, **(b)** B, **(c)** D **3.** $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

4. B **5.** a > 1 **6.** **(a)** D ; **(b)** A **7.** $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

8. **(a)** A, **(b)** 1210

9. **(a)** D, **(b)** (A)–(P),(R),(S) ; (B)–(Q),(S) ; (C)–(Q),(S) ; (D)–(P),(R),(S)

10. B **11.** 2 **12.** B **13.** C **14.** B